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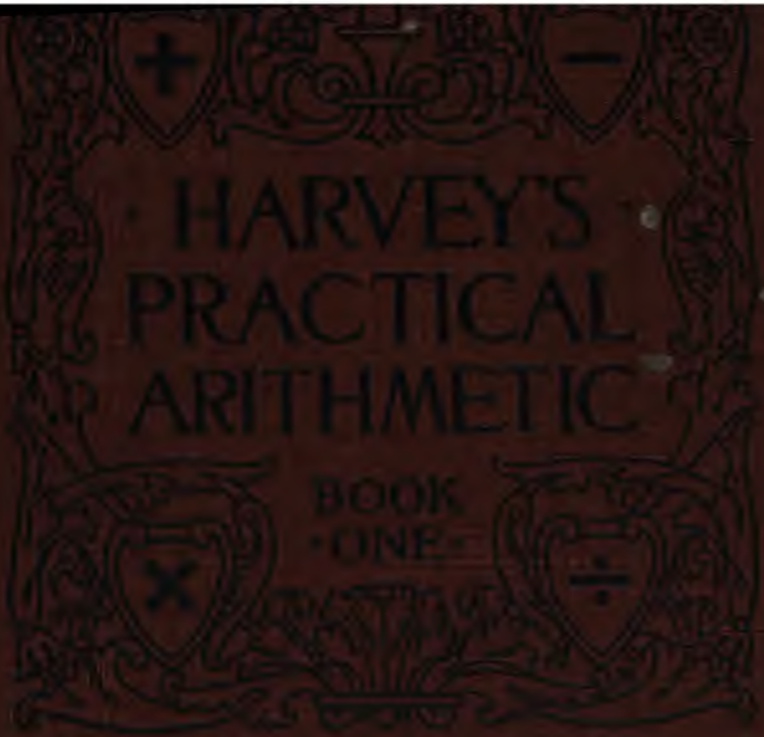

HARVEY'S PRACTICAL ARITHMETIC

BOOK ·ONE·

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PRACTICAL ARITHMETIC

BOOK I

BY

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HARVEY'S PRACT. AR. BOOK I.

E-P 1

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PREFACE

THIS series of arithmetics consists of two books. Book I presents matter that, under ordinary conditions, can be completed at the end of the fifth school year.

Book II presents material to meet all reasonable demands for work in arithmetic during the last three years of the elementary course.

Courses of study, the amount of time given to arithmetic in the program, and the quality of the instruction, vary so widely in different schools that it is impracticable to prescribe, in a series of arithmetics, the exact amount of work to be completed each year. It is thought best, however, to indicate the work that experience has shown may be done, under ordinary conditions, during each year of the course.

In many of the best systems of schools, no formal work in arithmetic is begun before the third year, while in other systems such work is begun earlier, though usually in a less formal way. Book I should not be put into the hands of pupils before the third year. If it is thought best to give arithmetical work earlier, the work in the first part of the book may be given by the teacher.

Each book is divided into three parts. Part I of the first book presents work that may be completed by the end of the third school year. Each of the remaining parts presents work for one year.

In this book the aim is to present exercises leading to a thorough mastery of the fundamental processes with whole numbers and of operations involving these processes in dealing with fractions, denominate numbers, and the elements of percentage. No attempt is made to teach the reasons for any process or operation, because the author believes that when the aim is to master either, no attempt should be made to teach the reason for it, unless the reason will aid in its mastery.

The illustrations are few in number, for the reason that the author believes that any legitimate purpose they may serve in an arithmetic may be better served, in most cases, by directing the pupil's attention to the objects the pictures represent, and by requiring such graphic work on the part of the pupil as he is capable of doing, when such work will contribute to his comprehension or to his application of arithmetical ideas. The pupil's work should be supplemented by such graphic work by the teacher as the given conditions may make necessary.

The reasoning called for by the work in Book I is that required in the solution of problems, and is adapted to the capacity of pupils for which the book is designed. It is to be distinguished from the work which the pupil does when the reasons for the operations are given by the book or by the teacher, and which he is expected to remember and repeat. The one is an exercise of the reason, the other an exercise of the memory.

In the arrangement of matter, the author has kept in mind what must be known and done in the mastery and application of any process or operation and has aimed to present such exercises, in such order as will most easily and readily lead to the desired skill on the pupil's part. These exercises are such as to bring out clearly the new thing at each stage of progress and to associate the new with the related old.

The aim in teaching arithmetic should be to develop the pupil's power to solve accurately and readily the arithmetical problems that arise in ordinary business transactions, and to secure such training in reasoning and in making correct and concise statements of arithmetical conditions, relations, and operations as may be properly associated with the work necessary for the development of this power.

Besides the writing and reading of numbers, the four arithmetical processes, — adding, subtracting, multiplying, and dividing, — are the only ones possible in the operations necessary for the solution of any arithmetical problem.

The solution of an arithmetical problem requires determining which of these processes shall be employed first, and with which of the given numbers, and the performance of the process; which next, and the performance of the process, and so on until the final result is found.

It is safe to say that more than two thirds of the time given to arithmetic by pupils during the six to eight years it is studied in the elementary schools is used in performing these processes, and yet comparatively few pupils in any of the grades of the elementary course habitually perform them accurately and rapidly.

By memorizing the separate facts which together constitute the addition, subtraction, multiplication, and division tables, and by appropriate and adequate drill exercises in the use of these facts, a thorough mastery of these four processes and their application in simple practical problems may be secured by the end of the fourth school year.

Such mastery of these processes will remove the necessity for a very large part of the "figuring" usually required throughout the upper grades, because an analysis of a problem, which gives each step in order, in the solution, may be made without actually adding, subtracting, multiplying, or dividing, by assuming each process to be performed as its necessity is shown in the progress of the analysis, and using a literal symbol, as "x," "y," or "z," to represent the result of the actual process. This kind of analysis gives all the training in reasoning possible in the solution of the problem, and the teacher knows that the pupil is able to secure the correct result readily because of his ability to perform the necessary processes. A further saving of time is effected whenever the pupil is required to find the correct answer, because he has been trained to perform these processes accurately and rapidly.

If a further reason is needed for the kind and amount of work, early in the course, which is necessary for the mastery of these basic processes, it is found in the fact that most people who make use of arithmetical operations in business have to perform operations of the same class day after day, where little reasoning is necessary, but where accuracy and rapidity are of the greatest importance.

Training in the statement of arithmetical facts, conditions, operations, and relations, correct in matter and form, should be carried on throughout the entire study of the subject, the requirements in this respect being adapted to the pupil's capacity. Definitions are statements of fact, and rules are statements of processes and of operations involving these processes. Unless the pupil can improve upon the form of these statements, he should learn them and make them a part of his arithmetical vocabulary.

L. D. HARVEY.

SUGGESTIONS

The first condition essential for the intelligent use of an arithmetic by pupils is, that they shall be able to read it intelligently. It is well, therefore, before pupils are assigned a lesson, especially in any part of the first book, to require an oral reading of what is to be assigned, and to question closely to see whether the matter read is understood; then to assign definitely what the pupil is to memorize and whatever else he is required to do.

If the pupils are trained to read the lessons readily, intelligently, and with good expression, the exercise has a distinct value as a reading lesson, paves the way for a thorough preparation of the lesson assigned, and materially increases their power to understand and use arithmetical language.

As an assistance to the teacher and pupil, new and unusual words and expressions are underscored. The teacher should be sure that the meaning of these words and expressions is understood by the pupils before they undertake the mastery of the lesson.

A new topic is frequently introduced by what is called the Study Recitation. The matter under this heading should be mastered by the pupils under the immediate direction and stimulation of the teacher. The teacher should always thoroughly master the treatment of the subject-matter under "Study Recitation" in advance, and should be able to give the illustrations and applications without reference to the book, and to extend or modify them as may be found necessary. These exercises are given where special preparation for further work by the pupils is deemed necessary. The time required for such preparatory work is time saved, even if an entire recitation period is necessary, because the pupils will do the following assigned work more intelligently, with greater interest, and more rapidly than would be possible without such preparation.

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BOOK ONE

PART I

COUNTING AND WRITING NUMBERS

1. *A*		*B*	
1	1 One	1, 1, 1, 1, 1, 1,	6 Six
1, 1	2 Two	1, 1, 1, 1, 1, 1, 1	7 Seven
1, 1, 1	3 Three	1, 1, 1, 1, 1, 1, 1, 1	8 Eight
1, 1, 1, 1	4 Four	1, 1, 1, 1, 1, 1, 1, 1, 1	9 Nine
1, 1, 1, 1, 1	5 Five	1, 1, 1, 1, 1, 1, 1, 1, 1, 1	10 Ten

To THE TEACHER. Place on the board the figures and numbers as arranged above.

Have pupils count the number of ones in each line in group *A* and in group *B*. When they have counted the number of ones in any line, have them make the figure or figures that stand for the number in that line.

Have pupils write the numbers from one to ten, and underneath each number, the figure or figures that represent it, as below.

One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten
1	2	3	4	5	6	7	8	9	10

Write the figures from 1 to 10 in irregular order on the blackboard, and have pupils name the number for which each figure stands.

Have pupils count the fingers on one hand; on both hands; the windows in the room; the panes of glass in one of the windows, and write the figure that stands for each number.

Place the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, on the board, where they may be left for pupils to copy, and have them copied until each pupil can make the figures well and rapidly.

When pupils can count groups of objects and write numbers to 10, place the following group of figures and numbers on the board, and have pupils count on from ten the number of ones in each line, and write the number as a word and also the figures that represent it.

1, 1, 1, 1, 1, 1, 1, 1, 1, 1   10 Ten

1	11 Eleven	1, 1, 1, 1, 1, 1	Sixteen
1, 1	12 Twelve	1, 1, 1, 1, 1, 1, 1	Seventeen
1, 1, 1	13 Thirteen	1, 1, 1, 1, 1, 1, 1, 1	Eighteen
1, 1, 1, 1	14 Fourteen	1, 1, 1, 1, 1, 1, 1, 1, 1	Nineteen
1, 1, 1, 1, 1	15 Fifteen	1, 1, 1, 1, 1, 1, 1, 1, 1, 1	Twenty

Write the numbers from 10 to 20 on the board and have pupils read, write, and count each number in order.

Write the words for the numbers from ten to twenty in irregular order and have pupils express each number in figures.

Drill pupils until they can count any group of objects up to twenty and can write the number in any group as a word and express it in figures.

Have pupils write any figure a given number of times on paper or on the board. As six 1's, three 8's, nine 2's, five 7's, etc.

Write two 3's in a line, as:	3, 3.	Have pupils count the number of 3's.
Write two more 3's under those first written, as:	3, 3.	
Write three 1's in a line, as:	1, 1, 1.	Have pupils tell how many 1's they have written.
Write three more 1's under those first written, as:	1, 1, 1.	
Write three more 1's, as:	1, 1, 1.	Have pupils tell how many 1's they have written.

So continue to 21.

In a similar way give drill in writing and counting other numbers.

Have pupils make dots, dashes, circles, squares, and other simple figures, arrange them in groups, count them, and write the number in each group.

Drill in naming in direct and reverse order the numbers 1 to 20 inclusive. Drill in writing these numbers in lines and in columns.

This work should be continued until pupils can readily count from 1 to 20 and from 20 to 1; can count the objects in any group less than 20, and can read and write numbers to 20 inclusive, and associate these numbers with the groups of objects which they number.

To the Teacher. When pupils have learned to count, read, and write numbers to 20, it is well to give them exercises in counting higher numbers. The numbers from 1 to 100 should be written on the board or on a chart, and left for the pupils to copy at their seats. In this way they will soon learn to count, read, and write numbers to 100.

Write the numbers as follows:

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

THE NUMBER 2

2. 1, 1 = how many 1's? How many are there here, 1, 1? Write the number two. 1 and 1 are how many?

$$1+1=2. \quad \text{Read} \begin{cases} \text{1 plus 1 equals 2.} \\ \text{1 and 1 are 2.} \end{cases}$$

The sign + is read **plus** and means that the numbers between which it stands are to be counted together, or added.

$$\begin{array}{r} 1 \\ +1 \\ \hline 2 \end{array} \quad \text{Read 1 and 1 are 2.}$$

The sign = is read **equal** or **equals** and means that the numbers between which it stands are *equal.*

Take in each hand 1 book, or 1 crayon, or 1 pencil, or 1 other object. How many have you in the left hand? How many in the right hand? Put both objects in the same hand. How many have you in that hand? One object (give the name) and one object (give the name) are two objects (give the name).

1. One pencil and one pencil are how many pencils? Write, 1 pencil + 1 pencil = 2 pencils. $1+1=2$.

2. One book + one book = how many books?
Write, 1 book + 1 book = 2 books. $1+1=2$.

3. Put 2 pieces of crayon on the desk; then take 1 away. How many are left? 1 crayon from 2 crayons leaves how many crayons?

Write and read, 2 crayons − 1 crayon = 1 crayon. $2-1=1$.

$2-1$ is read "2 minus 1" and means 2 less 1, or 1 from 2.

$2-1=1$ is read "2 minus 1 equals 1," or 2 less 1 is 1.

$2-2=0$ is read "2 minus 2 equals 0," or 2 less 2 is 0.

4. If I have 2 sticks in my hand and place 1 of them on the desk, how many are left in my hand? 1 stick from 2 sticks leaves how many?

Write and read, 2 sticks − 1 stick = 1 stick. $2-1=1$.

5. If I have 2 pennies and John takes 1 and Henry the other, how many have they taken? How many are left?

6. Two pennies from two pennies leaves how many?

7. 1 from 2 leaves how many? $2-1=?$ $\begin{array}{r}2\\-1\\\hline\end{array}$

8. 2 from 2 leaves how many? $2-2=?$ $\begin{array}{r}2\\-2\\\hline\end{array}$

TABLE

Learn: **1 + 1 = 2** **2 − 1 = 1**
2 − 2 = 0

$\begin{array}{r}2\\-1\\\hline 1\end{array}$ Read 1 from 2 leaves 1. $\begin{array}{r}2\\-2\\\hline 0\end{array}$ Read 2 from 2 leaves 0.

To the Teacher. Have pupil select objects and make similar statements concerning them.

Have them ask and answer similar questions.

Have one pupil ask the questions and others answer.

Have different forms of questions and statements, as in 1 to 8 above.

THE NUMBER 3

3. 1. 1, 1, how many 1's? Write the number.

2. 1, how many 1's? Write the number.

3. 1, 1, 1, how many 1's? Write the number.

4. 2 1's and 1 are how many?
2 and 1 are 3. $2+1=3$.

$$\begin{array}{r} 2 \\ +1 \\ \hline 3 \end{array}$$

5. 2 caps and 1 cap are how many caps?

6. 2 sticks and 1 stick are how many sticks?

7. 2 crayons and 1 crayon are how many crayons?

8. If John has 1 nickel and his father gives him 2 more, how many does he then have?

9. If I place 3 marks on the board and rub one out, how many are left?

10. 1 from 3 leaves how many?

11. One apple and two apples are how many apples?

12. One apple + two apples = how many apples?

13. If I have 3 dollars and give 2 dollars away, how many have I left?

14. If there are 3 erasers on the desk and John takes all three of them away, how many are left?

15. 3 dollars − 1 dollar = how many dollars?

16. Mary has 3 dollars and Jane has 1 dollar less than Mary. How many dollars has Jane?

17. 3 less 1 is how many?

18. If James earns 3 dimes and Henry earns 2 dimes, how many more does James earn than Henry?

19. 2 and 1 are how many?

20. 2 from 3 leaves how many?

21. $3-2=$?

To the Teacher. Have pupils use 3 objects, making similar questions and statements, and have the questions answered by the pupil who asks them, or by other pupils. Continue practice until pupils know that 2 and 1, or 1 and 2, counted or added together always give 3.

Table

Learn:

$3-1=2$

$2+1=3$ **$3-2=1$**

$3-3=0$

2 pints = 1 quart

To the Teacher. The teacher should illustrate, by means of pint and quart cups, the meaning of pint and quart, if pupils do not already know.

22. 1 and 1 are how many?

23. 1 from 2 leaves how many?

24. 2 less $2=$?

25. There are 2 pints in a quart. If I have a quart measure full of water and pour out 1 pint, how much is left?

26. 1 pint from 2 pints leaves what?

27. 1 pint from 1 quart leaves what?

28. Count and write the numbers from 20 to 30.

THE NUMBER 4

4.

TABLE

Learn:

$$\begin{array}{cc} 3+1=4 & \begin{array}{c}4-1=3\\4-3=1\end{array}\\ 2+2=4 & \begin{array}{c}4-2=2\\4-4=0\end{array}\end{array}$$

1. 1, 1, 1, 1, how many 1's? Write the number.

2. 3 1's and 1 are how many?.

3. 3 and 1 are how many? $3+1=?$

$$\begin{array}{r}3\\+1\\\hline\end{array}$$

4. 1, 1, how many 1's? Write the number.

5. 1, 1, 1, how many 1's? Write the number.

6. 1, 1, 1, 1, how many 1's? Write the number.

7. 2 1's and 2 1's are how many?
2 and 2 are how many? $2+2=?$

$$\begin{array}{r}2\\+2\\\hline\end{array}$$

8. 3 boys in the class may stand. 1 boy may stand. How many boys are now standing?

9. 3 boys and 1 boy are how many boys?

10. 4 boys are standing. 2 may be seated. How many are now standing? 2 boys from 4 boys leaves how many boys?

11. The 2 who were seated may stand. How many are now standing? 2 boys and 2 boys are how many?

12. 3 boys may be seated. How many are left standing? 3 boys from 4 boys leaves how many boys?

13. Mary had 4 cents and spent 2 cents for a pencil. How many had she left? 2 from 4 leaves how many?

14. If Mary spends 2 cents for a pencil and 1 cent for a stick of candy, how many cents does she spend for the pencil and the candy? 2 and 1 are how many?

Learn: **4 quarts = 1 gallon**

To THE TEACHER. The teacher should have a gallon and a quart measure, and by their use, give pupils a clear idea of what is meant by a gallon.

15. If I put 2 quarts of water in a gallon measure, how many more quarts must I put in to fill it?

16. If I take out of a gallon measure 2 pints, how many quarts do I take out? How many quarts remain?

17. 1 quart from 4 quarts leaves how many quarts? 1 from 4 leaves how many?

18. 1 quart from 1 gallon leaves how many quarts?

19. $\begin{array}{r} 2 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +1 \\ \hline \end{array}$ Give the sums.

20. $\begin{array}{r} 4 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -3 \\ \hline \end{array}$ Give the differences.

To THE TEACHER. Have pupils make statements and questions similar to those above, using objects up to 4. The teacher should write combinations on the board, as:

$$\begin{array}{r} 1 \\ 2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ 2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ 1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ 1 \\ \hline \end{array}$$

and point to the different combinations, requiring the pupils to give the sums instantly. Deal in the same way with the following, calling for differences:

$$\begin{array}{r} 4 \\ -2 \end{array} \quad \begin{array}{r} 3 \\ -1 \end{array} \quad \begin{array}{r} 4 \\ -1 \end{array} \quad \begin{array}{r} 2 \\ -1 \end{array} \quad \begin{array}{r} 4 \\ -3 \end{array} \quad \begin{array}{r} 2 \\ -2 \end{array}$$

21. Count and write the numbers from 30 to 40.

THE NUMBER 5

5. TABLE

Learn:

4 + 1 = 5 — **5 − 1 = 4**, **5 − 4 = 1**

3 + 2 = 5 — **5 − 2 = 3**, **5 − 3 = 2**

1. 1, 1, 1, 1, how many 1's? Write the number.
1, how many 1's? Write the number.
1, 1, 1, 1, 1, how many 1's? Write the number.
4 1's and 1 are how many?
4 and 1 are? 4 + 1 = ?

$$\begin{array}{r} 4 \\ +1 \\ \hline \end{array}$$

2. 1, 1, 1, how many 1's? Write the number.
1, 1, how many 1's? Write the number.
1, 1, 1, 1, 1, how many 1's? Write the number.
3 1's and 2 1's are how many 1's?
3 and 2 are how many? 3 + 2 = ?

$$\begin{array}{r} 3 \\ +2 \\ \hline \end{array}$$

3. A boy earns 5 cents selling newspapers. He spends 3 cents of his money for more newspapers. How many cents has he left? 3 from 5 leaves how many?

4. A man drives 5 miles in an hour and another man walks 2 miles in an hour. How much farther does the first man go than the second? 2 miles and how many miles are 5 miles? 2 and 3 are how many? 2 and how many are 5?

5. A boy has a nickel and buys 4 cents' worth of candy. How much change does he get?

6. If you spend 2 cents for candy and 1 cent for an apple, how many cents do you spend in all?

How much change should you get if you give the storekeeper a nickel?

7. A class recites five times a day. If it recites 3 times before noon, how many times must it recite after noon? If it recites one lesson before recess in the afternoon, how many lessons must it recite after recess?

8. 3 from 5 leaves how many?

9. 1 from 2 leaves how many?

10. 1 and 1 and 3 are how many? Show it by marks upon the blackboard; by objects on the desk.

11. How many are 2 and 2; 1 and 3; 2 and 3; 5 – 4; 4 – 2; 1 from 3; 3 less 2; 5 less 4; 3 and 2; 1 and 1; 2 and 2; 4 less 3; 5 less 5; 3 – 3; 5 less 3; 5 – 2; 5 – 1?

12.	5	4	2	3	1	2	5
	–2	–3	+2	+2	+1	+1	–5

To the Teacher. The teacher should have pupils ask similar questions, the other members of the class answering; no number larger than 5 to be used and no results larger than 5 required.

In drill with abstract numbers, pupils should be required to show that they understand what they are doing by illustrating with objects or putting together and separating numbers of objects.

13. Count and write the numbers from 30 to 50.

THE NUMBER 6

6. TABLE

Learn:

5 + 1 = 6 **6 − 1 = 5**
6 − 5 = 1

4 + 2 = 6 **6 − 2 = 4**
6 − 4 = 2

3 + 3 = 6 **6 − 3 = 3**

1. 1, 1, 1, 1, 1, how many 1's? Write the number.
1, how many 1's? Write the number.
1, 1, 1, 1, 1, 1, how many 1's? Write the number.

5 and 1 are? $5 + 1 = ?$

$$\begin{array}{r}5\\+1\\\hline\end{array}$$

2.
$$\begin{array}{rl}1,\ 1,\ 1,\ 1 & \text{How many 1's? Write the number.}\\ +\qquad 1,\ 1 & \text{How many 1's? Write the number.}\\\hline 1,\ 1,\ 1,\ 1,\ 1,\ 1 & \text{How many 1's? Write the number.}\end{array}$$

4 and 2 are how many? $4 + 2 = ?$

$$\begin{array}{r}4\\+2\\\hline\end{array}$$

3.
$$\begin{array}{rl}1,\ 1,\ 1 & \text{How many 1's? Write the number.}\\ 1,\ 1,\ 1 & \text{How many 1's? Write the number.}\\\hline 1,\ 1,\ 1,\ 1,\ 1,\ 1 & \text{How many 1's? Write the number.}\end{array}$$

3 and 3 are how many? $3 + 3 = ?$

$$\begin{array}{r}3\\+3\\\hline\end{array}$$

4. If I make 5 dots on the board and then 1 more, how many have I made in all? 5 and 1 are how many? If I rub out five dots, how many are left? How many

dots have I taken away from the 6? 5 from 6 leaves how many?

5. John's father gives him a nickel and he finds a cent in the street. How much money has he now? If he spends 3 cents for a top and hands the clerk the nickel to pay for the top, how many cents should he get back? How many will he have then? 3 from 5 leaves how many?

6. How many pints are there in a quart?

7. In 2 quarts there are how many pints? In 1 more quart there are how many pints?

8. How many pints are there in 3 quarts?

To the Teacher. This is not given as an example in multiplication, but for the purpose of having pupils add 2 and 2 and 2. Similar questions are given further on and for a similar purpose.

9. How many quarts are there in a gallon?

10. A gallon and 2 quarts equal how many quarts?

11. If there are 6 quarts of water in a pail, and I fill a gallon jug from it, how much water is left in the pail?

12. How many pints are there in 2 quarts?

13. $\begin{array}{r} 6 \\ -5 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -2 \\ \hline \end{array}$

14. $\begin{array}{r} 5 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +3 \\ \hline \end{array}$

Learn: **3 feet = 1 yard**

To the Teacher. Illustrate with foot rule and yardstick, so that pupils will understand thoroughly the meanings of these terms.

15. How many times must I lay off a foot measure to make 1 yard? To make 2 yards? Measure off 1 foot, 2 feet, 1 yard on the board, using a foot measure.

16. If I have a piece of cord 2 yards long, how many feet long is it? Draw a line 2 yards long on the blackboard. If I cut off a piece 1 yard and 1 foot long, how many feet do I cut off? Show it on the line drawn on the blackboard. What is the length of the piece left?

17. If you are in school 6 hours a day, and 3 hours of the time in the forenoon, how long should you be in school in the afternoon?

18.	5	3	2	4	2	2	3	1
	+1	+3	+2	+1	+1	+3	+1	+1

19.	5	6	5	5	3	4	5	2
	−2	−3	−1	−4	−2	+2	+1	+2

20.	4	5	3	6	6	4	6	2
	−2	−3	−1	−4	−2	−1	−5	+3

21. Count from 50 to 60 and write the numbers.

To the Teacher. Have pupils make and solve problems involving addition and subtraction of numbers with which they have already had practice. Have pupils write numbers between 1 and 50 from dictation. Do not dictate the numbers in order. Require figures to be made with care.

THE NUMBER 7

7.

TABLE

Learn : **6 + 1 = 7**

5 + 2 = 7

4 + 3 = 7

7 − 1 = 6
7 − 6 = 1
7 − 2 = 5
7 − 5 = 2
7 − 3 = 4
7 − 4 = 3

1. 6 and 1 are how many? Write the number. $7-1=?$ $7-6=?$ $5+1=?$ $6+1=?$

2. How many 1's must be added to 5 to make 7? 5 and how many are 7? 2 and how many are 7? 5 and 2 are how many? $5+2=?$

$$\begin{array}{r}5\\ +2\\ \hline\end{array}$$

3. If you have four marks on a piece of paper, how many more must you add to make 7 marks?

4. If there are 3 sticks in a pile on the desk, how many more must you add to make 7?
4 and 3 are how many? $4+3=?$

$$\begin{array}{r}4\\ +3\\ \hline\end{array}$$

5. There are 7 days in the week. How many working days are there? How many school days? How many more days are there in the week than there are working days? How many more days are there in the week than there are school days?

6. What is the *first* day of the week? The *second?* The *fourth?* The *seventh?* The *third?* The *sixth?* The *fifth?*

7. After the third day, how many more days are there in the week? After the fifth, how many days?

8. A man has 4 letters to mail. If it takes a 1-cent stamp for one of them and a 2-cent stamp for each of the others, how much will the stamps cost? *Ans.* 1 cent + 2 cents + 2 cents + 2 cents or —— cents.

If he pays for them with a nickel and pennies, how many pennies will it take?

9. Put 7 marks on paper or on the blackboard. Separate them into two groups, having 3 in one group. How many are there in the other?

If the larger group is erased, how many will be left?

10. How many yards are there in 6 feet? A stick is 7 feet long. How much more than 2 yards long is it?

11. A man wishes to buy 7 quarts of vinegar. The grocer has a gallon measure and a quart measure. How can he measure the 7 quarts and use both measures?

TO THE TEACHER. Have each pupil make 7 marks on the board. Then require each to erase any number of the marks he pleases and tell how many are erased and how many remain. Thus: I erased —— of the marks and —— marks are left; —— from 7 leaves ——. The blanks are to be filled with the proper numbers. Similar drill should be given with other numbers of marks as the work proceeds.

Have the pupils of the class stand and number in order; thus: 1, 2, 3, 4, 5, etc. Then have them number again in the same order, thus: first, second, third, fourth, fifth, etc.

12. Count the numbers from 60 to 100 and write the numbers from 60 to 70.

THE NUMBER 8

8.

Table

Learn:

7 + 1 = 8

8 − 1 = 7
8 − 7 = 1

6 + 2 = 8

8 − 2 = 6
8 − 6 = 2

5 + 3 = 8

8 − 3 = 5
8 − 5 = 3

4 + 4 = 8

8 − 4 = 4
8 − 8 = 0

What is the next number after 7?

1. If you have counted 7 marbles, how many more must you count to make 8 marbles?

2. Place 1 piece of crayon on the desk; place other pieces in a pile counting on from the 1 piece to 8 pieces. How many pieces are there in the pile?

3. 1 piece of crayon and —— pieces of crayon are 8 pieces.

7 and 1 are how many? 7 + 1 = ? $\begin{array}{r}7\\+1\\\hline\end{array}$

4. 6 and 1 and 1 are how many?

5. 8 is how many more than 6?

6. 6 and 2 are how many? 6 + 2 = ? $\begin{array}{r}6\\+2\\\hline\end{array}$

7. 5 and how many are 8? Show it by the use of objects.

8. Show how many will be left if 3 objects are taken away from 8 objects. State it.

5 and 3 are how many? $5+3=?$

$$\begin{array}{r}5\\ +3\\ \hline\end{array}$$

5 from 8 leaves how many? 3 from 8 leaves how many?

$8-5=?$ $8-3=?$

9. 2 and 2 are how many?

10. 4 and 2 and 2 are how many?

4 and 4 are —— ? $4+4=?$

$$\begin{array}{r}4\\ +4\\ \hline\end{array}$$

6 and 2 are —— ? $6+2=?$

$$\begin{array}{r}6\\ +2\\ \hline\end{array}$$

11. If there are 8 panes of glass in the window and 4 of these are in the upper sash, how many are in the lower sash? If 3 panes of glass were broken out of the window, how many whole panes would be left? If each pane cost 1 dime, how much would all the glass in the window cost? If each pane cost a quarter of a dollar, how many quarters would 8 panes cost?

12. 4 men are at work in a field. Each receives 1 dollar per day. How many dollars do they all receive for 1 day's work? How many days will they have to work to earn 8 dollars? 4 dollars and 4 dollars are how many dollars?

13. It takes 8 yards of cloth to make Helen's suit. If she gets two remnants one of which is 5 yards long, how long must the other remnant be?

14. Draw a line 6 inches long. If each inch of the line stands for 1 foot in length, for what length does the 6-inch line stand?

On the same scale, how long a line should you draw to show the length of a board 8 feet long?

15. A walks 3 miles an hour and B walks 4 miles an hour. If they walk in the same direction, how far apart will they be at the end of 1 hour? At the end of 2 hours? Show by lines on the board.

16. How far will each have walked in 2 hours? Show by use of drawings on the board.

Learn: **2 pints = 1 quart**

8 quarts = 1 peck

4 pecks = 1 bushel

TO THE TEACHER. Illustrate with pint, quart, and peck measures the relative capacities of the three measures. If you do not have measures in the schoolroom, it may be possible to get pupils to bring some of the more common measures to the school for the purpose of illustration. Do not use the liquid quart measure.

17. If a man has a peck of corn and sells 2 quarts, how many quarts has he left? If he receives 1 dime for each quart he sells, how many dimes does he receive? What he has left is worth how many dimes?

TO THE TEACHER. In conducting the following drills, the teacher may read the exercises, requiring pupils to give answers orally; the pupils may read the exercises silently or aloud, and give answer as each exercise is read.

The teacher may read each exercise and require pupils to write answers on paper as the teacher reads; the pupils may copy the exercises and write the answers for seat work.

Require the work to be done as rapidly as possible.

18. Drill:

$$\begin{array}{cccccc} 5 & 3 & 2 & & 6 & 7 \\ \underline{+2} & \underline{+} & \underline{+} & \underline{+3} & \underline{-} & \underline{-} \\ & 7 & 8 & 5 & 4 & 3 \end{array}$$

$$\begin{array}{ccccccc} 8 & 8 & 3 & 4 & 8 & 8 & 2 \\ \underline{-4} & \underline{-2} & \underline{+5} & \underline{+4} & \underline{-6} & \underline{-1} & \underline{+6} \end{array}$$

19. Add:

$$\begin{array}{cccccccc} 2 & 1 & 3 & 2 & 4 & 5 & 2 & 1 \\ 2 & 2 & 1 & 3 & 2 & 1 & 1 & 2 \\ 2 & 2 & 3 & 1 & 1 & 2 & 3 & 1 \\ \underline{2} & \underline{2} & \underline{1} & \underline{2} & \underline{1} & \underline{0} & \underline{2} & \underline{4} \end{array}$$

20. 8 less 2, less 2, less 2 is how many?

21. $8-3+2-4$ is how many?

22. $2+1+3-2+3+1=?$ $8-3-2+4-2=?$

Subtract:

23.

$$\begin{array}{ccccccc} 6 & 6 & 7 & 5 & 7 & 3 & 2 \\ \underline{-1} & \underline{-3} & \underline{-5} & \underline{-4} & \underline{-2} & \underline{-1} & \underline{-2} \end{array}$$

24.

$$\begin{array}{ccccccc} 8 & 8 & 7 & 6 & 5 & 4 & 5 \\ \underline{-5} & \underline{-2} & \underline{-3} & \underline{-2} & \underline{-3} & \underline{-2} & \underline{-2} \end{array}$$

To the Teacher. Have pupils drill each other on work like the preceding, reviewing daily the previous work.

25. Write the numbers from 70 to 80.

THE NUMBER 9

9. TABLE

Learn: **8 + 1 = 9** — **9 − 1 = 8**, **9 − 8 = 1**

7 + 2 = 9 — **9 − 2 = 7**, **9 − 7 = 2**

6 + 3 = 9 — **9 − 3 = 6**, **9 − 6 = 3**

5 + 4 = 9 — **9 − 4 = 5**, **9 − 5 = 4**

1. 8 and 1 are —— ? 8 + 1 = ? 9 − 1 = ? 9 − 8 = ?
   7 and 2 are —— ? 7 + 2 = ? 9 − 2 = ? 9 − 7 = ?

2. Show that 7 and 2 are 9 by counting a group of 7 objects with a group of 2 objects. Show how many are left if two objects are taken from nine objects; if seven objects are taken from nine objects. 9 − 2 = ? 9 − 7 = ?

3. What two numbers added make 9? What two other numbers? 6 and 3 are —— ? 6 + 3 = ?

Show by the use of objects that 6 objects and 3 objects = 9 objects.

TO THE TEACHER. First read the questions, and have pupils do what is required or answer the questions orally; then have pupils read and answer. Continue until answers can be given instantly.

4. 5 and 4 are how many? Make one group of 5 1's and another group of 4 1's. Count them together. How many 1's are there?

5 and 4 are —— ? 5 + 4 = ?
9 − 5 = ? 9 − 4 = ?

5. If I pay 2 cents a yard for ribbon, how much will 2 yards and 1 yard cost?

6. A boy has 9 quarts of oats in a bag. If he gives a peck of the oats to a horse, how many quarts remain?

7. There are 9 pupils in the arithmetic class. If all but 2 have their lessons, how many have their lessons?

8. How many pints are there in a quart? How many quarts are there in a gallon? In two gallons?

9. From a pail of water a man fills a pint cup twice and a gallon measure twice. How many quarts does he take out of the pail?

10. A man works 9 hours a day. If he works 5 hours before dinner, how many hours must he work after dinner?

11. If the school day is 6 hours long, and 3 hours of the time come before noon, how long is the afternoon session?

12. How many hours should a pupil be in school from Monday morning until Monday night? From Tuesday morning until Tuesday noon?

13. How many hours is it from noon until 5 o'clock in the afternoon? How many more hours is it until 9 o'clock at night?

TO THE TEACHER. If there is no clock in the room that the pupils can see, a drawing of the clock dial should be placed upon the blackboard and explained to the pupils.

14. From 2 o'clock in the afternoon, how many hours is it until 4 o'clock?

From 4 o'clock, how many hours is it until 8 o'clock in the evening?

How many hours is it from 2 o'clock to 8 o'clock?

15. Separate 9 objects into 2 groups; change objects from one group to the other until you have made all possible changes. Tell how many there are in each group after each change.

16. 2 + 2 + 4 + 1 = ? 9 − 4 − 2 − 1 − 2 = ?

17. 3 and 2 and 3 are how many? 8 − 3 − 2 − 2 = ?

18. 2 and 4 and 3 are how many? 9 − 3 − 4 − 2 = ?

19. 5 and 2 and 2 are how many? 9 − 5 − 2 − 2 = ?

20. 8 and 1 less 3 are how many? 9 − 8 + 3 − 1 = ?

21. 9 less 2 less 3 less 1 is how many? 2 + 3 − 1 + 5 = ?

22. A farmer buys 2 dollars' worth of nails, a shovel for 1 dollar, and an oil stove for 3 dollars. How many dollars does he pay for all?

23.	6	3	5	3	5	2	9	4	4
	+2	+6	+2	+4	+4	+7	−3	+2	+5

24.		2	4	9	8	7	5	9	9
	+3	+2	+4	−1	−7	−4	+3	−7	−4
	8								

25. 2 + 3 + 4 − 3 − 2 − 1 + 5 + 1 = ?

26. 5 + 4 − 6 + 5 − 4 − 2 + 7 − 5 = ?

27. Write numbers from 60 to 90.

THE NUMBER 10

10. TABLE

Learn:

9 + 1 = 10
10 − 1 = 9
10 − 9 = 1

8 + 2 = 10
10 − 2 = 8
10 − 8 = 2

7 + 3 = 10
10 − 3 = 7
10 − 7 = 3

6 + 4 = 10
10 − 4 = 6
10 − 6 = 4

5 + 5 = 10
10 − 5 = 5
10 − 10 = 0

1. Place 10 objects in one group. Separate these objects into two groups in as many ways as possible.

2. Each time you have the two groups tell how many objects you have in each and how many you have in both.

Name and write the numbers to fill the blanks:

3. 9 and — are 10.
4. 2 and — are 10.
5. 5 and — are 10.
6. 8 and — are 10.
7. 6 and — are 10.
8. — and 8 are 10.
9. — and 3 are 10.
10. — and 6 are 10.
11. 7 and — are 10.
12. 3 and — are 10.
13. 4 and — are 10.
14. 1 and — are 10.
15. — and 4 are 10.
16. — and 7 are 10.

17. How many cents are there in 1 dime?

18. If you have a dime and spend a nickel for candy, how many nickels have you left?

19. How many cents are there in a nickel?

20. How many nickels are there in a dime?

21. I have to pay a man 10 dollars. I have five-dollar, two-dollar, and one-dollar bills. If I give him a five-dollar bill, with what bills may I pay him the rest of the 10 dollars?

Learn:

10 cents = 1 dime
10 dimes = 1 dollar, or 100 cents
5 dimes = one half dollar, or 50 cents

22. If a boy travels from noon until 10 o'clock at night, how many hours does he travel?

23. How many hours less would he have traveled if he had stopped at 4 o'clock in the afternoon?

24. If 10 pupils are standing in a line and 3 of them are boys, how many are girls? If 6 of the 10 seat themselves, how many remain standing? If all are seated and 3 arise, how many remain seated?

25. 2 and 2 and 2 and 2 and 2 are how many?

26. 3 and 3 and 3 are how many?

27. 4 and 4 are how many?

28. Mary is 5 years older than Helen; Mary is 10 years old. How old is Helen?

29. Give sums and differences rapidly:

$$\begin{array}{rrrrr} 8 & 9 & 6 & 10 & 5 \\ -4 & -5 & +2 & -\ 6 & +5 \\ \hline \end{array}$$

$$\begin{array}{rrrrr} 9 & 2 & 8 & 4 & 9 \\ -7 & +3 & +2 & +4 & -3 \\ \hline \end{array}$$

30. Add:

$$\begin{array}{cccccc} 2 & 2 & 3 & & & \\ 2 & 5 & 2 & 1 & & \\ 3 & 1 & 3 & 4 & 3 & 1 \\ 1 & 1 & 1 & 3 & 3 & 5 \\ 2 & 1 & 1 & 2 & 4 & 4 \\ \hline \end{array}$$

$$\begin{array}{cccccc} 5 & 2 & 7 & 8 & 5 & 9 \\ 4 & 3 & 3 & 2 & 5 & 1 \\ \hline \end{array}$$

Add and subtract:

31. $4+5-3+2+2-5-4+3+2=?$

32. $2+3+4-7+6-4+2-5+9=?$

33. $6+4-8+7-5+3-2+6-4=?$

34. $10-2-2-2-2-2+3+3+3=?$

35. $10-3-3-3+4+4-6+4-2=?$

36. Write numbers from 90 to 100.

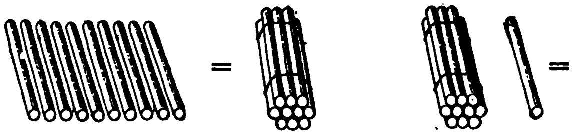
37. Write and read the numbers from 1 to 100 whose right-hand figure is 0.

To the Teacher. Pupils may give results rapidly with book before them, thus: nine, six, eight, ten, five, one, four, six.

Teacher may dictate the numbers and operations rapidly, and require pupils to give the final result.

UNITS AND TENS

11. 1. If I have a number of sticks on the table, by what figure shall I represent 1 of them?



2. If I tie 10 of them in a bundle, how shall I represent the 10?

3. How shall I represent the 10 in the bundle and 1 stick? It may be written, The 1 stands for the 1 stick, the 10 for the bundle of sticks. When counted together, they make 11.

$$\begin{array}{r}10\\ \underline{1}\\ 11\end{array}$$

4. Which of the 1's in 11 stands for the 1 stick? Which for the bundle of 10?

5. If there are 2 bundles of 10, what must I put in place of the 1 at the left? If there are 3 bundles? 4 bundles? 5 bundles? 6 bundles? 7 bundles? 8? 9?

6. If I have 1 bundle of sticks and 2 sticks and I count them together, how many have I? How do I write the number? How many does the first figure at the right stand for? How many does the figure at the left stand for? Both together?

7. If I have thirteen sticks, how shall I write the number? What does the 1 stand for? The 3?

8. If I have nineteen sticks, how shall I write the number? What does the 9 stand for? The 1?

When a figure stands alone, it tells how many 1's or units. When a figure stands at the left of another figure, it tells how many 10's.

9 tells how many units? 8 tells how many units?

9. If a 1 stands at the left of the 9 (19), 9 tells how many? The 1 tells how many?

10. If a 2 stands at the left of 8 (28), what does the 2 stand for? What does the 8 stand for?

11. How is twenty written? How many 10's are there in 20? How many units?

12. If you have 2 bundles with 10 sticks in each, how many sticks have you in both? If you have 1 more stick, how many have you? If you have 2 more? 3 more? 4 more? 5 more? 6 more? 7 more? 8 more? 9 more?

13. With 10 more sticks, you can make another bundle of 10. How many bundles have you then?

3 bundles of 10 sticks are thirty (30) sticks.
4 bundles of 10 sticks are forty (40) sticks.
5 bundles of 10 sticks are fifty (50) sticks.
6 bundles of 10 sticks are sixty (60) sticks.
7 bundles of 10 sticks are seventy (70) sticks.
8 bundles of 10 sticks are eighty (80) sticks.
9 bundles of 10 sticks are ninety (90) sticks.
10 bundles of 10 sticks are one hundred (100) sticks.

COUNTING, READING, AND WRITING TO 100

12. (1) 10
(2) 20
(3) 30
(4) 40
(5) 50
(6) 60
(7) 70
(8) 80
(9) 90
(10) 100

To the Teacher. The dots at the left should be placed upon the board and pupils taught to count them and to write the numbers at the right as given.

How many dots are there in each line?
How many dots are there in line 1?
How many dots are there in lines 1 and 2?
How many dots are there in lines 1, 2, 3?
How many dots are there in lines 1, 2, 3, 4?
How many dots are there in lines 1, 2, 3, 4, 5, 6, 7?
How many dots are there in lines 1, 2, 3, 4, 5, 6, 7, 8, 9, 10?

	Tens	Units
Twenty is written with a 2 and a 0.	2	0
Twenty-one is written with a 2 and a 1.	2	1

To the Teacher. Have pupils read the above statements and write the numbers as they read. Proceed in the same way with the numbers to thirty.

1. If you have made 10 marks, how many more must you make to have 20? 20 is how many more than 10?

2. If you have counted 20 marks, how many more must you count to count 30? 30 is how many more than 20?

3. 10 is one ten. 20 is how many tens? 30 is how many tens?

4. How many tens is 40? How many tens is fifty? Sixty? Seventy? Eighty? Ninety? One hundred?

5. Write each number from 20 to 40 and tell how it is written.

Seat and Board Work.

6. Copy the following table of numbers. Write the table from memory.

11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

To the Teacher. Place the above table on the board and drill pupils in reading the numbers as you point to them rapidly.

7. Write each number from 40 to 100 and tell how it is written.

8. Copy the following table, writing the numbers to fill the blanks:

ten	is	1 ten or 10
twenty	is	2 tens or 20
thirty	is	3 tens or 30
forty	is	4 tens or 40

fifty is 5 tens or —
sixty is 6 tens or —
seventy is 7 tens or —
eighty is 8 tens or —
ninety is 9 tens or —
one hundred is 10 tens or —

9. Write with figures 3 tens, 7 tens, 4 tens, 6 tens, 9 tens, 2 tens, 8 tens, 1 ten, 5 tens, 10 tens; twenty, fifty, thirty, seventy, ninety, sixty, ten, forty, eighty, one hundred.

10. Complete the table of numbers from 11 to 40 given in 5, by writing below it the numbers from 40 to 100.

11. Write the following numbers with figures, and write or tell the number of tens and units in each:

ILLUSTRATION: thirty-four 34 = 3 tens, 4 units.

twenty-three
forty-five
twenty-four
thirty-six
seventy-two
eighty-one
ninety-five
fifty-seven
eighty-eight
ninety-nine
eighty-five
sixty-seven
thirty-nine
forty-seven
fifty-six
eighty-two
seventy-three
twenty-eight
sixty-six
forty-four
fifty-four

Read the following numbers, and tell the number of tens and units in each:

ILLUSTRATION: 21 twenty-one, 2 tens, 1 unit.

12.	13.	14.	15.	16.
21	34	42	55	29
32	78	37	65	47
43	89	85	56	61
54	56	64	87	82
65	45	27	92	95
76	23	35	36	73
87	67	46	48	68
98	99	83	57	79
88	50	60	44	77
75	40	70	39	33
97	96	80	90	94

DRILL EXERCISE.

TO THE TEACHER. Dictate numbers and have pupils write them until they can write correctly, neatly, and rapidly in columns numbers less than 100.

Have one pupil dictate and the others write.

Write numbers less than 100 on the board and have pupils read the numbers as they are written.

Have one pupil write the numbers and others read.

Write many numbers on the board, point to different numbers rapidly, and have pupils read the numbers as you point to them.

Have pupils write all numbers from 10 to 100 whose unit figure is 2, 5, 3, etc.

WRITING AND READING NUMBERS FROM 100 TO 1000

13. Copy the following table, filling in the blanks:

	Hundreds Tens Units	
One hundred is written	1 0 0	1 hundred, no tens, no units.
Two hundred is written	2 0 0	2 hundreds, no tens, no units.
Three hundred is written	3 0 0	______________________.
Four hundred is written	4 0 0	______________________.
Five hundred is written	5 0 0	______________________.
Six hundred is written	6 0 0	______________________.
Seven hundred is written	7 0 0	______________________.
Eight hundred is written	8 0 0	______________________.
Nine hundred is written	9 0 0	______________________.
One thousand is written	1 0 0 0	______________________.

1. Write with figures: one hundred, four hundred, three hundred, five hundred, nine hundred, seven hundred, two hundred, six hundred, eight hundred, one thousand, forty, sixty, thirty, fifty, ninety, seventy, twenty, ten, eighty, two hundred seventy, five hundred thirty, three hundred ninety, six hundred eighty, four hundred sixty, nine hundred forty.

2. Read the following numbers: 20, 40, 300, 70, 400, 50, 800, 60, 700, 10, 900, 200, 80, 100, 1000, 90, 56, 87, 29, 30, 48, 52, 63, 74, 21, 82, 70, 400, 50, 63, 48, 27, 65.

Read the following numbers from left to right; from right to left; up; and down:

3.

Hundreds	Tens	Units
	1	0
1	1	0
2	1	0
3	1	0
7	5	0

4.

Hundreds	Tens	Units
	2	0
1	2	0
2	2	0
4	2	0
8	4	0

5.

Hundreds	Tens	Units
	3	0
1	3	0
2	3	0
5	3	0
4	2	0

6.

Hundreds	Tens	Units
	4	0
1	4	0
2	4	0
6	4	0
6	5	0

7.

Hundreds	Tens	Units
	5	0
1	5	0
2	5	0
7	5	0
8	3	0

8.

Hundreds	Tens	Units
	6	0
1	6	0
2	6	0
8	6	0
2	2	0

9.

Hundreds	Tens	Units
	7	0
1	7	0
2	7	0
9	7	0
1	4	0

10.

Hundreds	Tens	Units
	8	0
1	8	0
2	8	0
6	8	0
7	7	0

11.

Hundreds	Tens	Units
	9	0
1	9	0
2	9	0
4	9	0
1	4	0

12.

Hundreds	Tens	Units
1	0	0
3	0	0
5	0	0
8	0	0
7	0	0

13. Write with figures the following: forty, one hundred fifty, two hundred ten, five hundred fifty, sixty, seven hundred eighty, three hundred ninety, one hundred sixty, four hundred seventy. Tell the number of hundreds, tens, and units in each number.

Read the following numbers:

14. 56 78 92 34 83

15. 156 178 192 134 183

16. 47 52 23 37 68

17. 147 152 123 137 168

18. Write the numbers in the first line, placing a 2 at the left of each number and read. ILLUSTRATION: 256 (read, two hundred fifty-six).

19. Write the numbers in the first line again, placing in hundreds' place each of the figures 3, 4, 5, 6, 7, 8, 9, and read the resulting numbers.

Read the following numbers:

20. 156 220 350 475 802

21. 509 387 602 527 496

22. 300 750 90 190 230

23. Write the following numbers: one hundred seventy-five, three hundred twenty, five hundred sixty-five, seven hundred eighty-one, four hundred thirty-one, eight hundred forty-six, six hundred seventy-two, four hundred forty-six.

24. How many tens are there in each number above in 14 to 22? When there are no tens or units in a number, what figure is written in tens' place? In units' place?

Read the following numbers from left to right; from right to left; down; up.

	25.	**26.**	**27.**	**28.**	**29.**
1.	101	102	103	104	105
2.	201	202	203	204	205
3.	401	402	403	404	405
4.	901	902	903	904	905
5.	301	502	603	804	305
6.	601	302	703	704	805
7.	701	802	803	504	505

	30.	**31.**	**32.**	**33.**	**34.**
8.	106	107	108	35	64
9.	206	207	208	425	864
10.	406	407	408	307	428
11.	906	907	908	813	130
12.	706	807	808	720	270
13.	904	203	402	704	856
14.	605	701	803	992	994

TO THE TEACHER. When pupils can read the above and similar numbers correctly and rapidly, read the numbers to them and have them written on paper or blackboard. Then dictate miscellaneous numbers from 1 to 1000.

If written on paper, the errors may be found by reading the figures in order in each number, and having pupils mark errors.

ILLUSTRATION: Suppose the first number read is 402. Teacher or some pupil reads four, naught, two, or 4 hundreds, no tens, 2 units.

Read in the same way the figures in the remaining numbers.

UNITED STATES MONEY

14. The Coins: cent, nickel, dime, quarter dollar, half dollar, dollar.

1. How many cents are there in 3 dimes?

2. How many cents are there in 4 dimes?

3. How many cents are there in 5 dimes?

4. We have a silver coin worth 50 cents. What else is it called?

5. How many dimes are there in a half dollar? In 60 cents? In 70 cents? In 80 cents? In 90 cents? In 100 cents?

6. How many cents are there in a dollar?

7. How many cents are there in 2 halves of a dollar?

8. What is the coin next smaller in value than the half dollar? How many such coins equal a half dollar? How many cents are there in one of them?

9. What is the coin next smaller in value than a quarter of a dollar? How many such coins equal a dollar?

10. What is the coin next smaller in value than a dime? How many such coins equal a dime? How many equal 2 dimes? 3 dimes? 4 dimes? 5 dimes?

To the Teacher. A supply of toy money may be used by the class to advantage in this work. A substitute for toy money may be obtained by marking circles, the size of the different coins, on heavy paper, and marking the value of each plainly. Have pupils cut out the circles.

THE NUMBER 11

15. TABLE

Learn:

9 + 2 = 11 **11 − 9 = 2**
11 − 2 = 9

8 + 3 = 11 **11 − 8 = 3**
11 − 3 = 8

7 + 4 = 11 **11 − 7 = 4**
11 − 4 = 7

6 + 5 = 11 **11 − 6 = 5**
11 − 5 = 6

1. A dime = how many cents?

2. A dime and 1 cent = how many cents?

3. If 1 cent is taken away from 11 cents, how many are left?

10 + 1 = ? 10 and 1 are —— ? $\begin{array}{r}10\\+1\\\hline\end{array}$

9 and 2 are —— ? 9 + 2 = ? $\begin{array}{r}9\\+2\\\hline\end{array}$

4. Teacher should have pupils show by use of objects that

8 and 3 are 11, 7 and 4 are 11, 6 and 5 are 11.

5. If a boy is 11 years old to-day, how long since was he 8 years old?

6. 8 dollars and 3 dollars are how many dollars?

7. 11 birds were sitting on a tree; 6 of them flew away. How many remained?

8. If you have one dime and one cent, how many cents have you?

9. Harold is 6 years old. How much older is his sister Edith, who is 11 years old?

10. Draw a square 2 inches on each side and a triangle 1 inch on each side. What is the distance around each of them? What is the distance around both of them? How much further is it around the square than around the triangle?

11.	8	6	4	6	9	7	5
	+ 3	+ 5	+ 3	+ 3	+ 2	+ 4	+ 5
12.	8	4	2	3	10	6	7
	+ 2	+ 4	+ 5	+ 3	− 7	+ 4	− 2
13.	2	3	8	9	6	8	11
	+ 7	+ 8	+ 2	− 4	− 4	− 4	− 4

14. $2 + 2 + 3 = ?$

15. $3 + 3 + 3 + 2 = ?$

16. $11 - 3 - 3 - 3 = ?$

17. $2 + 2 - 3 + 4 + 2 - 5 = ?$

18. $4 + 6 - 4 - 3 + 2 + 1 = ?$

To the Teacher. Have the pupils make practical problems and drill each other in rapid additions and subtractions, no result being greater than 11.

THE NUMBER 12

16.

TABLE

Learn:

6 + 6 = 12 **12 − 6 = 6**

9 + 3 = 12 **12 − 9 = 3**
12 − 3 = 9

8 + 4 = 12 **12 − 8 = 4**
12 − 4 = 8

7 + 5 = 12 **12 − 7 = 5**
12 − 5 = 7

1. 9 and 3 are —— ? $9 + 3 = ?$ $\begin{array}{r} 9 \\ +\ 3 \\ \hline \end{array}$

2. 8 and 4 are —— ? $8 + 4 = ?$ $\begin{array}{r} 8 \\ +\ 4 \\ \hline \end{array}$

3. 7 and 5 are —— ? $7 + 5 = ?$ $\begin{array}{r} 7 \\ +\ 5 \\ \hline \end{array}$

4. 6 and 6 are —— ? $6 + 6 = ?$ $\begin{array}{r} 6 \\ +\ 6 \\ \hline \end{array}$

5. 12 less 9 are —— ? $12 - 9 = ?$ $\begin{array}{r} 12 \\ -\ 9 \\ \hline \end{array}$

6. 12 less 5 are —— ? $12 - 5 = ?$ $\begin{array}{r} 12 \\ -\ 5 \\ \hline \end{array}$

7. 12 less 7 are —— ? $12 - 7 = ?$ $\begin{array}{r} 12 \\ -\ 7 \\ \hline \end{array}$

8. In 1 week and 5 days, how many days are there?

9. 1 peck and how many quarts equal 12 quarts?

Learn: **There are 12 months in 1 year**
There are 12 things in 1 dozen
There are 6 things in a half dozen
There are 12 inches in 1 foot
There are 6 inches in a half foot

10. If a boy buys a half dozen marbles and some one gives him 4 more, how many does he then have? How many more must he have to make a dozen?

11. If plums are worth 1 cent each, how many cents will a half dozen cost? A dozen?

12. If I buy 5 cents' worth of plums, how much more must I pay to get a dozen if plums are worth 1 cent each? How many more plums do I get? How many do I get for 5 cents?

13. If I buy a peck of peas, how many more quarts must I buy to have a dozen quarts?

14. In 2 yards how many feet are there?

15. I have a stick 2 yards in length, and lay off a flower bed twice the length of the stick. How many yards long is the flower bed? How many feet long?

16. Draw, on the board, a line 1 foot long.

Draw another line a half foot long.

How many inches long is each line?

Draw a line 3 inches long. How much longer than the 3-inch line is each of the other lines?

17. Make a line a foot long on the board and mark the inches.

Make other lines of different lengths in inches and tell which ones together make a line 1 foot long.

18. In 2 yards and 1 foot, how many feet are there? In 3 yards? In 3 yards and 2 feet?

19. If there are twelve trees in a garden, and 5 of them are peach trees, and the rest are apple trees; —— Complete the problem by asking a question.

20. In a class of 12 pupils, 8 of them are more than 10 years of age; —— Complete the problem by asking a question.

21. August is the eighth month of the year. How many months are there from the end of August to the end of the year?

22.

$$\begin{array}{rrrrrrrrr} 8 & 5 & 6 & 3 & 9 & 10 & 5 & 11 & 12 \\ +4 & +5 & +2 & +8 & -5 & -6 & -5 & -3 & -4 \\ \hline \end{array}$$

23.

$$\begin{array}{rrrrrrrrr} 4 & 3 & 8 & 10 & 9 & 12 & 12 & 2 & 3 \\ +4 & +5 & +2 & +2 & +3 & -2 & -9 & +7 & +4 \\ \hline \end{array}$$

24. $2+2+2+2+2+2=$ how many? How many 2's $=12$?

25. $3+3+3+3=$ how many? How many 3's $=12$?

26. $4+4+4=$ how many? How many 4's $=12$?

27. If I take 4 from 12, how many are left?
If I take 4 from 8, how many are left?
If I take 4 from 4, how many are left?
How many times can 4 be taken from 12?

28. How many ones equal 2 half dozen?

29. $6 + 5 - 2 - 3 - 4 + 7 = ?$

30. $2 + 3 + 2 - 4 + 3 - 2 - 3 + 7 + 4 = ?$

31. $12 - 8 + 3 + 2 - 6 + 4 = ?$

32. $9 + 3 - 6 + 6 - 8 - 2 = ?$

33. $12 - 7 - 4 + 9 + 2 - 6 = ?$

To the Teacher. Provide the pupils with strips of stiff paper 1 inch wide and of different lengths to 1 foot. Mark the lengths in inches on each strip, and drill pupils in putting together strips of different lengths, and giving length and width of combined strips. Work in taking apart the joined strips, or in subtracting, should also be given.

Seat Work. Add the following and read the answers:

34.	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
31	56	285	124	432	247	852
56	31	714	835	657	351	646
87						

Subtract the following and read the answers:

41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**
47	98	973	857	729	958	906
35	65	421	613	319	213	503
12						

THE NUMBER 13

17.

TABLE

Learn:

9 + 4 = 13
13 − 9 = 4
13 − 4 = 9

8 + 5 = 13
13 − 8 = 5
13 − 5 = 8

6 + 7 = 13
13 − 7 = 6
13 − 6 = 7

1. 10 and 3 are —
10 + 3 = ?

$$\begin{array}{r}10\\+3\\\hline\end{array}$$

3. 8 and 5 are —
8 + 5 = ?

$$\begin{array}{r}8\\+5\\\hline\end{array}$$

2. 9 and 4 are —
9 + 4 = ?

$$\begin{array}{r}9\\+4\\\hline\end{array}$$

4. 7 and 6 are —
7 + 6 = ?

$$\begin{array}{r}7\\+6\\\hline\end{array}$$

5. A dozen and 1 more are how many?

6. Make a problem using a dozen as applied to objects.

7. A boy buys 2 loaves of bread for 10 cents and cakes for 2 cents; —— Complete the problem. He gives the baker 4 coins in payment. What are they?

8. How much more than a week are 13 days?

9. 8 and 5 are how many? 6 and 7? 9 and 4? 13 less 9? 8 from 13? 6 from 13? 5 from 13?

10. Name the different sets of two numbers which added together give 13.

11.

$$\begin{array}{r}13\\-8\\\hline\end{array}\quad\begin{array}{r}6\\+7\\\hline\end{array}\quad\begin{array}{r}9\\+4\\\hline\end{array}\quad\begin{array}{r}6\\+3\\\hline\end{array}\quad\begin{array}{r}2\\+7\\\hline\end{array}\quad\begin{array}{r}10\\-5\\\hline\end{array}\quad\begin{array}{r}12\\-7\\\hline\end{array}\quad\begin{array}{r}13\\-7\\\hline\end{array}$$

12.
$$\begin{array}{rrrrrrrr} 12 & 9 & 8 & 5 & 4 & 3 & 3 & 13 \\ \underline{-6} & \underline{-4} & \underline{-4} & \underline{+5} & \underline{+7} & \underline{+9} & \underline{+7} & \underline{-5} \end{array}$$

13. $13-2-3-4+2+1=?$

14. $2+5+4-3-5-2=?$

15. $3+1+5+3-4-6=?$

16. $3+3+3+3+1=?$

17. $6+6+1=?$

SEAT WORK. **18.** Complete the sets of two numbers which, added together, give the following numbers: 6, 7, 9, 8, 11, 13, 10, 12. Thus:

$$9=\begin{cases}6+3\\1+8\\5+4\\2+7\end{cases}\quad 6=\begin{cases}4+\\5+\\3+\\1+\end{cases}\quad 7=\begin{cases}5+\\6+\\4+\\2+\end{cases}\quad 8=\begin{cases}4+\\6+\\3+\\1+\end{cases}$$

$$11=\begin{cases}8+\\2+\\5+\\4+\\1+\end{cases}\quad 13=\begin{cases}5+\\2+\\7+\\4+\\10+\end{cases}\quad 10=\begin{cases}2+\\5+\\3+\\4+\\7+\end{cases}\quad 12=\begin{cases}9+\\7+\\4+\\2+\\6+\end{cases}$$

Add:

19.	**20.**	**21.**	**22.**	**23.**	**24.**
146	642	459	241	235	437
823	357	320	657	743	542

THE NUMBER 14

18. TABLE

Learn:

$9 + 5 = 14$ $14 - 9 = 5$ $14 - 5 = 9$

$8 + 6 = 14$ $14 - 6 = 8$ $14 - 8 = 6$

$7 + 7 = 14$ $14 - 7 = 7$

1. 10 and 4 are — $10 + 4 = ?$ $\begin{array}{r} 10 \\ +4 \\ \hline \end{array}$

2. 9 and 5 are — $9 + 5 = ?$ $\begin{array}{r} 9 \\ +5 \\ \hline \end{array}$

3. 8 and 6 are — $8 + 6 = ?$ $\begin{array}{r} 8 \\ +6 \\ \hline \end{array}$

4. 7 and 7 are — $7 + 7 = ?$ $\begin{array}{r} 7 \\ +7 \\ \hline \end{array}$

5. How many days are there in 2 weeks?

6. A dozen and how many more make 14?

7. 1 peck and how many quarts equal 14 quarts?

$ means dollars.

8. A man works 5 days and earns \$2 each day. How many dollars does he earn? $\$2 + \$2 + \$2 + \$2 + \$2 = ?$

How many days must the man work to earn 4 dollars?

How many days must he work to earn 14 dollars in all?

9. A man travels 6 miles an hour. How far does he go in 2 hours?

10. A man has to travel 14 miles. He travels 6 miles an hour for 2 hours. Ask a question to complete the problem.

Make a different problem using different numbers.

Teacher illustrate the making of the problem.

11. A man pays 6 dollars for a cord of wood and 7 dollars for a ton of coal. How much does he pay for both? How much would 2 tons of coal cost? How much more would 2 tons of coal cost than the ton of coal and the cord of wood?

12. Mary is 9 years old to-day. In how many years will she be 14 years old? If she is 3 years older than Jane, how old is Jane? How old will Jane be when Mary is 14?

13. In a pasture there are 14 horses. 6 of them are taken out for work. Complete the problem.

14.

$$\begin{array}{rrrrrrrr} 8 & 6 & 9 & 7 & 14 & 13 & 14 & 12 \\ +5 & +8 & +5 & +7 & -6 & -5 & -8 & -7 \\ \hline \end{array}$$

15.

$$\begin{array}{rrrrrrrr} 13 & 14 & 7 & 8 & 9 & 6 & 5 & 12 \\ -7 & -7 & +6 & +3 & +5 & +6 & +4 & -5 \\ \hline \end{array}$$

16. $3+4+7-4-5=?$

17. $2+8+4-6-3=?$

18. $6+8-7+4-5=?$

19. $3+2+4+3-8=?$

20. $2+5+7-3-2=?$

21. $8+6-3-5-4=?$

22. $4+5+3+2-8=?$

23. $9+5-4-3-5=?$

Learn:

24.	**25.**
4 + 2 = 6	4 + 3 = 7
14 + 2 = 16	14 + 3 = 17
24 + 2 = 26	24 + 3 = 27
34 + 2 = 36	34 + 3 = 37
44 + 2 = 46	44 + 3 = 47
54 + 2 = 56	54 + 3 = 57
64 + 2 = 66	64 + 3 = 67
74 + 2 = 76	74 + 3 = 77
84 + 2 = 86	84 + 3 = 87
94 + 2 = 96	94 + 3 = 97

SEAT WORK. **26.** Write all the sets of two numbers which, added together, make the following numbers: 8, 9, 10. When written, repeat from memory.

8 = {

9 = {

10 = {

Subtract:

27.	**28.**	**29.**	**30.**	**31.**	**32.**
849	937	689	578	847	983
623	215	432	145	214	872

33. Name all sets of 2 numbers which, added together, make 4, 5, 6, 7, 8, 9.

THE NUMBER 15

19. TABLE

Learn:

$9+6=15$ $15-6=9$
$15-9=6$

$8+7=15$ $15-7=8$
$15-8=7$

1. A nickel and a dime equal how many cents?

2. How many working days are there in a week? In two weeks?

3. 2 boys pick strawberries at 1 cent a quart; one picks 8 quarts and the other 7 quarts. How many cents does each receive? How much money do they both receive? How much more does one receive than the other? If they received 15 cents between them, how many quarts did they pick? If one boy picked 6 quarts, how many quarts must the other have picked?

4. It takes 15 hours to make a journey between two points. After a man has traveled 11 hours, how many hours more has he to travel?

5. How many nickels equal 15 cents?

6. How many quarts does a gallon measure hold? From a jar containing 15 quarts of water, a gallon measure is filled three times. How many quarts are taken out? How many quarts remain in the jar?

7. How many times can I take 4 from 15, and how many will be left?

8. How much will 3 pencils cost at a nickel each?

9. How many days are there in 2 weeks? In 2 weeks and 1 day?

10. 1 week and how many days equal 15 days?

11. How many feet equal 1 yard? Equal 3 yards?

12. I have a piece of rope 15 feet long. If I cut off 3 yards, how many feet remain? If I had cut off only 2 yards, how many feet would have been left?

To the Teacher. Have pupils make and answer similar questions.

13. A boy has a dozen marbles. How many more must he buy to have 15?

14. If he has 15 and gives away a half dozen, how many has he left?

15. A peck and how many quarts are equal to 15 quarts?

16. A pane of glass measures a foot by 15 inches. How much longer is it than it is wide?

17. Draw an outline of the pane of glass on the board.

18. A man buys 15 dollars' worth of goods and pays for it with 2 bills. What is the value of each bill?

19. Draw a line 15 inches long, marking off the inch spaces on the line; separate the line into two parts and tell how many inches there are in each part. Make other divisions into two parts, giving the number of inches in each part.

20. Draw a square 15 inches on each side. If the square is drawn on a scale of 1 inch to the foot, how large a piece of ground will the square represent? Give length and width.

21. Draw a line through the square so as to mark off one piece of ground 6 feet by 15 feet. What is the width of the other piece? Draw another line so as to mark off the square into four oblongs and give the length and width of each piece of ground.

To the Teacher. Give the pupils such assistance as may be needed, and such additional exercises as may be necessary to an understanding of the work.

22.	7	6	15	15	15	6
	+8	+9	−6	−8	−10	+7
23.	8	4	12	5	13	5
	+5	+9	−6	+6	−6	+10
24.	4	6	5	8	14	14
	+5	+3	+7	+6	−9	−7
25.	5	4	3	7	6	14
	+5	+4	+3	+7	+6	−8
26.	15	15	13	11	15	15
	−7	−4	+2	+4	−13	−11
27.	15	12	15	14	15	15
	−12	+3	−9	+1	−3	−5

28. 3 + 3 + 3 + 3 + 3 = ? How many 3's are there in 15? How many times can you take 3 away from 15?

29. 5 + 5 + 5 = how many? How many 5's are there in 15? How many times can you take 5 away from 15?

30. If you had 15 cents and gave each of two boys a nickel, how much money have you left?

Learn:

5 + 2 = 7 **5 + 3 = 8**
15 + 2 = 17 **15 + 3 = 18**
25 + 2 = 27 **25 + 3 = 28**
35 + 2 = 37 **35 + 3 = 38**
45 + 2 = 47 **45 + 3 = 48**
55 + 2 = 57 **55 + 3 = 58**
65 + 2 = 67 **65 + 3 = 68**
75 + 2 = 77 **75 + 3 = 78**
85 + 2 = 87 **85 + 3 = 88**
95 + 2 = 97 **95 + 3 = 98**

31. Add by 3's from 3 to 15. How many 3's are there in 15?

32. Add by 5's from 5 to 15. How many 5's are there in 15?

33. Name the sets of two numbers which added together make 10, 9, 8.

SEAT WORK. **34.** Write all the sets of two numbers which, added together, make the following. When written, repeat from memory.

$$11 = \left\{ \vphantom{\begin{matrix}a\\b\\c\end{matrix}} \right. \qquad 12 = \left\{ \vphantom{\begin{matrix}a\\b\\c\end{matrix}} \right. \qquad 13 = \left\{ \vphantom{\begin{matrix}a\\b\\c\end{matrix}} \right.$$

THE NUMBER 16

20. TABLE

Learn: $9 + 7 = 16$ $16 - 7 = 9$

$16 - 9 = 7$

$8 + 8 = 16$ $16 - 8 = 8$

1. How many 8's = 16?

2. How many times can you fill a peck measure from 16 quarts of clover seed?

3. How many times can you take 2 from 16? How many 2's equal 16?

4. If from a piece of rope 16 feet long I cut off 3 yards, how long a piece is left? How many yards can I cut off from what is left? What will be the length of the remaining piece?

5. Draw a line on the board 16 inches long and let each inch stand for 1 foot. How many feet does the whole line stand for? Draw a mark across the line showing where you would cut off what would stand for 3 yards. Draw another line where you would cut off what would stand for the next 2 yards. What is the length of the line left?

6. If William buys apples for 9 cents and candy for 7 cents, how much do the apples and candy cost him? He has dimes, nickels, and cents with which to pay for what he has bought. How many and what coins can he use in making exact payment?

7. Count by 4's from 4 to 16. How many 4's = 16?

8. Count by 8's from 8 to 16. How many 8's = 16?

9. $\begin{array}{r} 9 \\ +7 \\ \hline \end{array}$ $\begin{array}{r} 8 \\ +8 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -9 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -8 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -7 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -10 \\ \hline \end{array}$

10. $\begin{array}{r} 11 \\ +4 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -5 \\ \hline \end{array}$ $\begin{array}{r} 14 \\ -8 \\ \hline \end{array}$ $\begin{array}{r} 7 \\ +9 \\ \hline \end{array}$ $\begin{array}{r} 12 \\ -4 \\ \hline \end{array}$ $\begin{array}{r} 12 \\ +3 \\ \hline \end{array}$

11. $\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$ $\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$ $\begin{array}{r} 8 \\ +2 \\ \hline \end{array}$ $\begin{array}{r} 9 \\ -5 \\ \hline \end{array}$ $\begin{array}{r} 8 \\ -6 \\ \hline \end{array}$ $\begin{array}{r} 9 \\ -2 \\ \hline \end{array}$

12. How many inches equal a foot? 16 inches = how many inches more than 1 foot?

13. A dime is how much less than 16 cents? Than 14 cents?

14. 2 nickels and how many cents equal 16 cents? 14 cents? 15 cents?

15. $4+8+3-7+8-9-5+4+2=?$

16. $3+5+7-6+5-7-4+9+4=?$

17. To each of the following numbers add 7; then 4; 5; 3; 6; 2.

9, 6, 2, 7, 8, 4, 3, 5, 1

To the Teacher. Work on 17 may be both oral and written. One pupil may give orally the sum of 7 and each of the numbers, thus: sixteen, thirteen, nine, fourteen, fifteen, eleven, ten, twelve, eight. Another pupil may add 4 to the numbers, and so on.

Different pupils may, one after the other, give all the sums to see who can give them all correctly in the shortest time. Place the name of the pupil making the best record on the board, with his record. Let the name remain until another pupil makes a better record. Pupils may write the sums on paper at seats or in class. Drill should be continued until all the sums can be given correctly in 30 seconds.

THE NUMBER 17

21. TABLE

Learn : $9+8=17$ $17-9=8$

$17-8=9$

1. How many feet = 3 yards?

2. 3 yards and how many feet = 17 feet?

3. If a pupil has to read 17 pages during the day, and reads 5 pages before recess in the forenoon and 4 pages between recess and noon, how many pages does he read in the forenoon? How many pages has he to read in the afternoon?

4. 10 from 17 leaves how many?

5. 11 and how many equal 17?

6. 5 and how many equal 17?

7.	17	8	17	8	7	9	17
	−9	+9	−8	+8	+9	+4	−8

8.	15	12	14	7	17	15	17
	−6	−6	−7	+8	−10	−1	−12

9. $2+2+5+6=$?

10. $8+7-6-4+3=$ how many?

11. 2 5's $+3+2+1-8=$ how many?

12. 2 8's $-9=$ how many?

13. 2 7's $+2-8=$ how many?

14. 2 6's $+5-9=$ how many?

15. 3 4's $+3+2=$ how many?

Make problems calling for the following sums and differences:

16. $9+8=?$ $\quad 17-8=?$ $\quad 17-9=?$

17. $8+9=?$ $\quad 8+4+5=?$ $\quad 12+5=?$

18. $16-8=?$ $\quad 15-7=?$ $\quad 6+11=?$

19. A boy is 8 years older than his sister who is 7 years old. How old is he? Ask and answer another question using the numbers given.

How old will each be in 2 years? How old was each 4 years ago? What was the sum of their ages 4 years ago?

20. A grocer gives a customer 17 cents in change from the cash drawer. With what coins can he pay him?

SEAT WORK. **21.** Fill the blanks:

$$16=\begin{cases}8+?\\12+?\\14+?\\10+?\\3+?\end{cases}\qquad 15=\begin{cases}?+9\\12+?\\5+?\\?+?\\?+?\\10+?\\4+?\end{cases}\qquad 14=\begin{cases}?+10\\9+?\\?+?\\7+?\\?+?\\2+?\end{cases}$$

$$13=\begin{cases}5+?\\?+?\\?+?\end{cases}\qquad 12=\begin{cases}2+?+?\\?+?+4\\?+3+?\\?+?+?\\4+?+?\end{cases}\qquad 11=\begin{cases}8+?\\2+?\\?+7\\7+?\\5+?\end{cases}$$

22. Write the following numbers, and add each set of two:

two hundred seven	eight hundred five
four hundred two	one hundred four
three hundred six	five hundred two
six hundred three	nine hundred five

23. Write the following numbers and subtract numbers in line *b* from those in line *a*; numbers in line *d* from those in line *c*:

a.	eight hundred fifty-six	nine hundred eighty-seven
b.	three hundred five	four hundred twelve
c.	six hundred ninety-five	seven hundred eighty-nine
d.	two hundred three	three hundred twenty-three

24. How many pints = 1 quart? How many quarts = 1 gallon? How many quarts = 1 peck? How many pecks = 1 bushel? How many inches = 1 foot? How many feet = 1 yard? How many things = 1 dozen? A half dozen? How many cents = 1 nickel? 1 dime? 1 quarter dollar? 1 half dollar?

How many cents = 1 dollar? Dimes = 1 dollar? Quarters = 1 dollar? Halves = 1 dollar? Dimes = a half dollar? Nickels = a dime? Nickels = a quarter? Nickels = a half dollar?

TO THE TEACHER. Ask the above questions and have pupils answer orally. Have pupils write the answers in the form of tables.

THE NUMBER 18

22. TABLE

Learn : **9 + 9 = 18** **18 − 9 = 9**

1. How many 9's = 18?

2. How many times can you take 9 from 18?

3. If a floor is 18 feet long, how many strips of carpet a yard wide laid across the floor would it take to cover it?

TO THE TEACHER. Draw the floor on a scale of 1 inch to a foot, and have pupils draw the lines showing where the strips join.

4. If the floor is 15 feet wide, how many strips of carpet a yard wide, laid lengthwise of the floor, will it take to cover it? Draw the floor on a scale of 1 inch to a foot, and draw lines showing where the strips join.

5. If I buy a dozen and a half eggs at 1 cent an egg, how many cents will they cost?

6. If I pay for them with a dime, nickel, and cents, how many cent pieces will it take?

7. 18 is how many more than a dozen? How many more than a half dozen? How many half dozen = 18?

8. Mary is 9 years old and John is twice as old. How old is John?

9.	9	18	9	7	6	9	18	17
	+9	−9	+8	+8	+9	+4	−8	−9

10.	7	9	6	4	3	8	13	17
	+7	+8	+5	+6	+7	−5	−8	−8

11.	8	3	15	6	8	14	16	15
	+7	+9	−8	−4	+8	−5	−9	−7

12. $18 - 9 - 4 - 2 =$ how many?

13. $7 + 8 - 5 + 3 =$ how many?

14. $2 + 6 + 9 - 7 =$ how many?

15. $8 + 4 -$ a half dozen = how many?

16.	9	7	8	5	6	4	3	2
	+9	+7	+8	+5	+6	+4	+3	+2

17. 2 8's = ? 2 9's = ? 2 4's = ? 2 6's = ?
2 1's = ? 2 7's = ? 2 5's = ? 2 3's = ?

18. 15 is how many 5's? How many 3's?

19. 12 is how many 2's? How many 3's? How many 4's?

20. 10 is how many 5's? How many 2's?

21. 8 is how many 4's? How many 2's?

22. 9 is how many 3's? 10 is how many 5's?

23. 6 is how many 3's? 4 is how many 2's?

24. 2 4's are how many 2's? 3 4's are how many 6's?

25. 4 4's are how many 8's? A dozen and a half are how many 9's?

26. Add 2's from 2 to 18; to 50. Add 3's from 3 to 36.

DRILLS

ADDITION DRILL

23.

DRILL 1

Give the sums rapidly from left to right and from right to left:

1.	1+2	2+2	3+3	4+4	5+5	6+6	7+7
2.	1+3	2+3	3+4	4+5	5+6	6+7	7+8
3.	1+4	2+4	3+5	4+6	5+7	6+8	7+9
4.	1+5	2+5	3+6	4+7	5+8	6+9	
5.	1+6	2+6	3+7	4+8	5+9		
6.	1+7	2+7	3+8	4+9			
7.	1+8	2+8	3+9		8+8	9+9	
8.	1+9	2+9			8+9		

TO THE TEACHER. This table should be placed on the board. The teacher should point to different combinations, requiring pupils to give sums. The work must be done rapidly to be of any value; pupils should know the sums, and not find them by counting. If a pupil does not know, require him to commit to memory a certain number each day, until all are mastered.

DRILL 2

Name the missing number:

ILLUSTRATION: 8 + ? = 15. Say, 8 + 7 = 15.

1. 1 + ? = 2; 2 + ? = 4; 3 + ? = 6; 4 + ? = 8; 5 + ? = 10; 6 + ? = 12; 7 + ? = 14; 8 + ? = 16; 9 + ? = 18
2. 1 + ? = 4; 2 + ? = 5; 3 + ? = 7; 4 + ? = 9; 5 + ? = 11; 6 + ? = 13; 7 + ? = 15; 8 + ? = 17
3. 1 + ? = 5; 2 + ? = 6; 3 + ? = 8; 4 + ? = 10; 5 + ? = 12; 6 + ? = 14; 7 + ? = 16.

4. 1 + ? = 6; 2 + ? = 7; 3 + ? = 9; 4 + ? = 11; 5 + ? = 13; 6 + ? = 19.

5. 1 + ? = 7; 2 + ? = 8; 3 + ? = 10; 4 + ? = 12; 5 + ? = 14.

6. 1 + ? = 8; 2 + ? = 9; 3 + ? = 11; 4 + ? = 13.

7. 1 + ? = 9; 2 + ? = 10; 3 + ? = 12.

8. 1 + ? = 10; 2 + ? = 11.

SUBTRACTION DRILL

24. DRILL 3

Subtract each number in each column from the number at the left of the column:

2	10	7	8	9	5	9
1	6	2	7	12	8	7
7	8	0	4	3	7	15
4	4	5	9	13	4	14
10	3	3	10	5	3	5
12 – 8	14 – 9	11 – 9	15 – 11	16 – 15	17 – 13	18 – 4
5	7	6	12	8	6	3
3	5	8	14	6	9	6
6	11	4	13	4	14	13
9	13	10	6	7	15	16
11	2	1	5	10	12	12
12	12	11	3	11	11	11

TO THE TEACHER. With books open, require individual pupils to give in order, and as rapidly as possible, the difference between each number in a column and the number at the left of the column.

Drill until each pupil can give all the differences in the table in less than one minute. When any pupil can do this, excuse him from further drill on the table.

Pupils may copy the table on paper, writing the differences as follows:

$$12 - \left\{ \begin{array}{l} 2 = 10 \\ 1 = 11 \\ 7 = \ 5 \\ 4 = \ 8 \\ 10 = \ 2, \text{ etc.} \end{array} \right.$$

DRILLS—SUMS OF TWO DIGITS

25. In the following, think the sum and give the unit figure in each sum; thus, $\begin{array}{c} 3 \\ \underline{8} \end{array}$. Think the sum and say 1.

Drill 4

1.	3	9	4	9	3	8	5	8
	$\underline{8}$	$\underline{9}$	$\underline{8}$	$\underline{8}$	$\underline{7}$	$\underline{8}$	$\underline{7}$	$\underline{0}$
2.	9	6	6	7	4	7	9	6
	$\underline{6}$	$\underline{8}$	$\underline{6}$	$\underline{5}$	$\underline{6}$	$\underline{9}$	$\underline{0}$	$\underline{7}$
3.	9	3	8	9	4	5	0	5
	$\underline{5}$	$\underline{9}$	$\underline{7}$	$\underline{2}$	$\underline{9}$	$\underline{8}$	$\underline{5}$	$\underline{5}$
4.	8	4	7	7	5	6	7	4
	$\underline{2}$	$\underline{7}$	$\underline{6}$	$\underline{7}$	$\underline{6}$	$\underline{4}$	$\underline{0}$	$\underline{4}$

Drill 5

In the following, state first the units' figure in the sum, and then the sum; thus $6+8$, say 4, 14. $7+9$, say 6, 16.

5. $6+8$ $\quad 5+9$ $\quad 6+4$ $\quad 2+9$ $\quad 3+4$ $\quad 6+6$ $\quad 4+4$ $\quad 3+6$

6. 7+9 8+2 4+7 3+8 2+3 7+7 2+2 1+7

7. 8+7 7+6 3+9 5+3 5+2 9+9 3+3 8+9

8. 5+8 8+4 9+4 8+8 2+7 9+1 1+6 7+3

9. 7+5 5+6 5+4 6+2 5+5 2+4 9+6 1+8

Drill 6

To the Teacher. Drill the pupils on the numbers in the columns *A*, *B*, *C*, *D*, *E*, *F*, *G*, by naming any number and requiring pupils to give the first number larger having a given units' figure; the nearest number smaller having a given units' figure, thus: take the number 44; give the first number larger having the units' figure 6; 2; 8; 3; 7; 1; 0. Give the nearest number smaller having the units' figure 2; 8; 3; 5; 7; 6; 9; 1. This drill should be continued until pupils can tell the larger or smaller numbers without reference to the numbers in the columns.

A	*B*	*C*	*D*	*E*	*F*	*G*
10	23	36	49	62	75	88
11	24	37	50	63	76	89
12	25	38	51	64	77	90
13	26	39	52	65	78	91
14	27	40	53	66	79	92
15	28	41	54	67	80	93
16	29	42	55	68	81	94
17	30	43	56	69	82	95
18	31	44	57	70	83	96
19	32	45	58	71	84	97
20	33	46	59	72	85	98
21	34	47	60	73	86	99
22	35	48	61	74	87	100

1. Add: $\begin{array}{r}51\\ \underline{6}\end{array}$ What is the units' figure in the sum? What is the first number beyond 51 which has 7 for a units' figure? The number is 57.

2. Add: $\begin{array}{r}57\\ \underline{6}\end{array}$ What is the units' figure? The next number having this units' figure? What is the sum of 57 and 6?

3. Add: $\begin{array}{r}63\\ \underline{9}\end{array}$ Units' figure? Next number with this units' figure? Sum?

4. Add: 72 Units' figure? Add: 46 Units' figure?
7 Sum? 5 Sum?

5. Add: 79 Units' figure? Add: 56 Units' figure?
8 Sum? 8 Sum?

6. Add: 87 Units' figure? Add: 64 Units' figure?
5 Sum? 5 Sum?

DRILL 7

To the Teacher. **Drill pupils in adding 7 to each of the numbers in line 1 below beginning with 36, thus: The units' figure in the sum of 7 and 6 is 3; 7 added to 36 gives the first number larger than 36 having 3 for a units' figure, or 43. 7 added to 56 gives 63, 7 and 26 are 33, 7 and 46 are 53, etc.**

When pupils understand the mode of procedure, have them give the sums without naming the numbers added, thus: forty-three, sixty-three, thirty-three, etc.

Continue drill by having each of the numbers from 1 to 10 added to the numbers in line 1. Proceed in the same way with the numbers in the other lines.

Vary the drill by having each of the numbers from 1 to 10 added to the numbers under *a*, *b*, *c*, etc.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1.	36	56	26	46	96	66	86	16	76
2.	45	65	85	25	95	75	15	35	55
3.	24	44	64	94	74	34	84	14	54
4.	63	23	43	13	53	93	73	33	83
5.	42	72	62	32	12	52	82	92	22
6.	11	31	91	21	81	61	41	71	51
7.	37	87	27	67	17	97	57	77	47
8.	18	38	58	98	28	88	68	48	78
9.	39	19	49	79	29	99	69	89	59

To THE TEACHER. Drill pupils in adding the numbers in columns, thus: Beginning with 9 in the units' column under *a*, add the numbers in that column successively; to this sum add the numbers in the units' column under *b*, *c*, etc. until the sum is at least 100.

Vary the drill by beginning with any number less than 9 and then adding as before. Continue the drill until pupils can add accurately and rapidly to 100. Time them for results.

In beginning this exercise, place several figures in a column on the board, thus:

$$\begin{array}{rr} 8 & 37 \\ 7 & 29 \\ 9 & 22 \\ 5 & 13 \\ 8 & \\ \hline 37 & \end{array}$$

Ask pupil to give sum of 8 and 5. When given, write 13 at the right of the 5. Ask for units' figure in sum of the right-hand figure in 13, or 3, and 9. When given, ask for first number larger than 13 having 2 for the units' figure. Write 22 at right of the 9. Ask for units' figure in sum of 2 and 7. Ask for first number larger than 22 having 9 for the units' figure. When given, write 29 at the right of 7. Ask for units' figure in sum of 9 and 8. Ask for first number larger than 29 having 7 for the units' figure. Write 37 at the right of 8. Draw a line underneath the column added, and write the sum below.

Continue similar work on the board until pupils have mastered the two steps, then have them add, writing only the last sum.

DRILL IN SUBTRACTING

26. Subtracting any number less than 10 from any number between 10 and 100.

DRILL 8

To THE TEACHER. Give drill in subtracting each of the lower numbers from the number represented by the figure above it, or if that number is the smaller, from that number increased by 10. Give the nearest number having this difference for its units' figure, which is smaller than the upper number.

ILLUSTRATIONS

1. $\begin{array}{r} 34 \\ -6 \\ \hline \end{array}$ 4 is less than 6. Subtract 6 from $4 + 10$, or 14. The difference is 8. The nearest number below 34 having 8 for its units' figure is 28. $34 - 6 = 28$.

2. $\begin{array}{r}38\\-3\\\hline\end{array}$ 8 is more than 3. 3 from 8 is 5. The nearest number below 38 having 5 for its units' figure is 35. $38-3=35$.

3. $\begin{array}{r}22\\-8\\\hline\end{array}$ 2 is less than 8. Subtract 8 from 12. The difference is 4. The nearest number below 22 having 4 for its units' figure is 14. $22-8=14$.

4. $\begin{array}{r}53\\-7\\\hline\end{array}$ 7 from 13 is 6. The nearest number below 53 having 6 for its units' figure is 46. $53-7=46$.

5. $\begin{array}{r}29\\-3\\\hline\end{array}$ 3 from 9 is 6. The nearest number below 29 having 6 for its units' figure is 26. $29-3=26$.

6. $\begin{array}{r}76\\-9\\\hline\end{array}$ 9 from 16 is 7. The nearest number below 76 having 7 for its units' figure is 67.

7. $\begin{array}{r}64\\-6\\\hline\end{array}$ 6 from 14 is 8. The difference is 58.

8. $\begin{array}{r}56\\-8\\\hline\end{array}$ 8 from 16 is 8. The difference is 48.

9. $\begin{array}{r}97\\-4\\\hline\end{array}$ 4 from 7 is 3. The difference is 93.

10. $\begin{array}{r}95\\-9\\\hline\end{array}$ 9 from 15 is 6. The difference is 86.

11. $\begin{array}{r}80\\-8\\\hline\end{array}$ 8 from 10 is 2. The difference is 72.

12. $\begin{array}{r}71\\-9\\\hline\end{array}$ 9 from 11 is 2. The difference is 62.

13. $\begin{array}{r}50\\-8\\\hline\end{array}$ 8 from 10 is 2. The difference is 42.

TO THE TEACHER. Pupils should be drilled until the units' figure and the required number come to the mind as a single mental operation.

Drill 9

To the Teacher. Have pupils subtract numbers from 1 to 10 from the numbers given in this table.

10	30	60	40	70	20	80	50	90
21	81	11	71	61	31	41	51	91
22	52	32	72	62	92	42	12	82
43	23	73	53	83	63	13	33	93
64	44	54	84	24	14	34	94	74
55	65	85	15	45	75	95	25	35
76	86	96	36	16	46	26	56	66
97	77	47	67	37	87	57	27	17
88	18	28	98	58	38	68	48	78
39	69	19	29	89	59	79	49	99

Drill 10

Beginning with 3, add by 3's to 36.
Beginning with 4, add by 4's to 48.
Beginning with 5, add by 5's to 60.
Beginning with 6, add by 6's to 72.
Beginning with 7, add by 7's to 84.
Beginning with 8, add by 8's to 96.
Beginning with 9, add by 9's to 108.
Beginning with 10, add by 10's to 120.

Reverse the process by subtracting each number from 2 to 10 successively from 12 times itself.

Thus, beginning with 24, subtract by 2's to 0; beginning with 36, subtract by 3's to 0; beginning with 48, subtract by 4's to 0, etc.

ADDITION

27. To add numbers of two or more figures when the sum of any column is ten or more.

Study Recitation

1. Add:

$$\begin{array}{r} 74 \\ \underline{68} \\ 142 \end{array}$$

8 and 4 are 12. Write 2, the units' figure of the sum, under the 8, and add 1, the tens' figure, to the next column. 1 and 6 and 7 are 14. Write 4, the first figure in the sum, at the left of the 2 and under 6, and 1 at the left of the 4.

2. Add:

$$\begin{array}{r} 359 \\ 269 \\ \underline{532} \end{array}$$

2, 9, and 9 are 20. Write the 0 under 2. Add the 2 to the next column. 2, 3, 6, and 5 are 16. Write the 6 under 3, add 1 to the next column. 1, 5, 2, and 3 are 11. Write 11 at the left of 6.

WRITTEN

Add the following:

3.	4.	5.	6.	7.
56	42	493	792	527
23	75	846	487	843
75	29	275	369	657

8. In a school 23 of the pupils are girls and 19 are boys. How many pupils are there in the school?

$$\begin{array}{r} 23 \\ \underline{+19} \end{array}$$

STATEMENT. The number of pupils in the school is the sum of 23 pupils and 19 pupils, or —— pupils.

9. A boy spends 35 cents for a reader, 25 cents for a speller, and 10 cents for a copy book. How many cents does he spend for all?

Statement. He spends for all the sum of 35 cents, 25 cents, and 10 cents, or —— cents.

10. A train of cars goes 38 miles the first hour, 49 miles the second hour, and 44 miles the third hour. How far does it go in the three hours?

Statement. The train goes in 3 hours the sum of 38 miles, 49 miles, and 44 miles, or —— miles.

11. There are 48 desks in one schoolroom and 54 desks in another schoolroom. How many desks are there in both rooms?

Statement. In both rooms there are 48 desks + 54 desks, or —— desks.

12. A merchant buys from one farmer 28 pounds of butter; from another 36 pounds; from a third 48 pounds. How many pounds does he buy from the three?

Statement. He buys from the three farmers the sum of 28 pounds, 36 pounds, and 48 pounds, or —— pounds.

13. If I buy 4 quarts of milk for 24 cents, and 2 pounds of butter for 58 cents, what is the entire cost?

Statement. The entire cost of the milk and butter is the sum of 24 cents and 58 cents, or —— cents.

Add:

14.	**15.**	**16.**	**17.**	**18.**
256	493	127	325	278
324	257	890	268	139
152	842	374	423	456
537	756	259	157	234
245	239	462	623	562

SUBTRACTION

28. To subtract a number of two or more figures from another number, when some of the figures in the subtrahend have a greater value than the corresponding figures in the minuend.

Learn:

The **subtrahend** is the number to be subtracted.

The **minuend** is the number from which the subtrahend is to be subtracted.

The **difference** or **remainder** is the result of the subtraction.

$$\begin{array}{rl} 246 & \text{Minuend} \\ 132 & \text{Subtrahend} \\ \hline 114 & \text{Difference,} \\ & \text{or} \\ & \text{Remainder} \end{array}$$

Study Recitation

Subtract:

1.
$$\begin{array}{r} 42 \\ -27 \\ \hline 15 \end{array}$$

7 cannot be taken from 2. Add 10 to 2, making 12. 7 from 12 = 5. Write 5, the difference between 7 and 12, below the 7. Add 1 to 2, the next figure in the subtrahend, making 3. Subtract 3 from 4, the number above it. Write 1, the difference, below the 2.

2.
$$\begin{array}{r} 435 \\ -276 \\ \hline 159 \end{array}$$

6 cannot be taken from 5. Add 10 to 5, making 15. 6 from 15 = 9. Write 9 below the 6. Add 1 to 7, making 8. 8 cannot be taken from 3. Add 10 to 3, making 13. 8 from 13 = 5. Write 5 below the 7. Add 1 to 2, making 3. 3 from 4 = 1. Write 1 below the 2.

3.
$$\begin{array}{r} 504 \\ -278 \\ \hline \end{array}$$

8 cannot be taken from 4. What must be done? 8 must be taken from what number? What is done with the difference? What must be done next? What number must be taken from 10? What is done with the difference? What must be done next?

4.

$$\begin{array}{r} 430 \\ -178 \\ \hline 252 \end{array}$$

What must be done to 0? After subtracting 8, what must be done to 7? What must then be done to 3? After the second subtraction, what must be done to 1? What must be done then?

Learn: 10 *must be added to a figure in the minuend when it is less in value than the number to be subtracted from it.*

When 10 *is added to any figure in the minuend,* 1 *must be added to the next figure in the subtrahend.*

TO THE TEACHER. At this stage do not attempt to teach the reason for adding 10 to a figure in the minuend, and 1 to the next figure in the subtrahend. Later, pupils will be able to discover the reason themselves, when they understand that adding equal numbers to two numbers does not affect the difference. If 10 is added to any figure in the minuend, and 1 is added to the figure of the next higher order in the subtrahend, both have been increased by the same number, and the difference is the same.

If preferred, instead of adding 1 to a figure in the subtrahend, it may be taken from the next figure in the minuend, though the plan illustrated is less likely to lead to mistakes on the pupils' part.

Learn: If the subtrahend and difference are added, their sum is the same as the minuend.

In examples 1, 2, 3, and 4 point out the subtrahend, the minuend, the difference or remainder.

See whether your subtraction is correct by adding the difference to the subtrahend in each case. The sum should be the same as the minuend.

WRITTEN

Subtract:

5.	**6.**	**7.**	**8.**	**9.**
468	523	428	642	275
273	216	254	375	138

10.	**11.**	**12.**	**13.**	**14.**
429	654	493	827	427
273	279	276	348	237

15.	**16.**	**17.**	**18.**	**19.**
642	278	147	200	427
354	129	98	157	309

20. In the first five grades in a school there are 238 pupils; in the first four grades there are 196 pupils. How many pupils are there in the fifth grade?

STATEMENT. In the fifth grade there are 238 – 196, or —— pupils.

21. How many more pages or less pages are there in your arithmetic than in your reader?

22. There are 30 days in September, 31 days in October, and 30 days in November. How many days are there in the three months?

23. There are 31 days in December, 31 days in January, and 28 days in February. How many days are there in these three months?

24. There are 31 days in March, 30 days in April, and 31 days in May. How many days are there in these three months?

25. There are 30 days in June, 31 days in July, and 31 days in August. How many days are there in these three months?

26. There are 365 days in a year. If school is in session 176 days in a year, how many days during the year is there no school?

27. If James buys a reader for 35 cents, and an arithmetic for 36 cents, and gives the merchant a dollar bill in payment, how much change should he get?

28. The minuend is 38, the subtrahend is 17. What is the difference?

29. Which is the smaller, the minuend or the subtrahend?

30. The difference between two numbers is 8; the smaller number is 4. What is the larger number? Which is the minuend? Which is the subtrahend?

31. The sum of two numbers is 24. One of them is 8, what is the other? Which of the three numbers is the minuend? Which is the subtrahend?

32. What is a minuend? A subtrahend? The difference or remainder?

33. The sum of the ages of a father and his son is 54 years; the son is 16 years old. What is the age of the father?

34. The sum of three numbers is 256; the sum of two of them is 118. What is the other number?

35. A man earns $ 35 in January and $ 36 in February. If he spends $ 48 for board and clothes and $ 5 for incidentals, how many dollars has he left?

UNITED STATES MONEY

29. To write, read, add, and subtract dollars and cents.

The dollar sign, $, placed before a number, means the same as the word "dollar" or "dollars" written after the number. Thus, $ 25 is read 25 dollars.

When dollars and cents are written as one number, a period is placed between the dollars and cents. Thus, $ 15.75 is read fifteen dollars seventy-five cents.

Read: $10.15; $35.10; $46.27; $32.12; $156.15.

When cents are written alone, two figures are used with a . and the $ sign at the left. Thus, $.25 is read twenty-five cents; $ .37 is read thirty-seven cents.

If the number of cents is less than ten, it is written thus, $.01, $.02, $.03, $.04, $.05, $.06, $.07, $.08, or $.09, and read one cent, two cents, three cents, etc.

When numbers representing dollars are written in columns, the sign is placed before the top number only.

Read the following. Add:

1.	2.	3.	4.	5.
$25.15	$156.20	$.15	$1.10	$2.00
18.25	37.50	27	2.05	8.00
17.75	56.38	35	3.07	12.00
207.12	86.12	04	8.20	17.08

6. A man earns $ 15.75 during the week. He spends $ 4.50 for board and $ 5.65 for other purposes. How much does he spend? How much has he left?

MULTIPLICATION AND DIVISION — TABLE OF TWOS

Study Recitation

30. 1. If apples cost 2¢ apiece, how many cents will 4 apples cost? ¢ means cent or cents. 1¢ is read 1 cent; 2¢ is read two cents. We have learned how to find how much the 4 apples will cost by addition.

$$\begin{array}{r} 2¢ \\ 2¢ \\ 2¢ \\ \underline{2¢} \\ 8¢ \end{array}$$

2¢ + 2¢ + 2¢ + 2¢ = 8¢. If the apples had cost 3¢ apiece, we should add 3¢ + 3¢ + 3¢ + 3¢ = 12¢.

2. How many times 2¢ is 8¢? How many times 2 is 8?

3. How many times 3¢ is 12¢? How many times 3 is 12?

If we remember that 4 times 2 is 8, and that 4 times 3 is 12, we can answer the questions above without taking the trouble to add.

Answer questions 4 and 5 without adding:

4. If I walk 2 miles an hour, how far can I walk in 4 hours?

5. If I pay 3¢ apiece for pencils, how many cents will 4 pencils cost?

6. If I pay 5¢ apiece for pads, how many cents will 2 pads cost?

7. If Ruth is 4 years old and Edith twice as old, how old is Edith?

If the same number is to be taken several times, we can shorten our work by multiplying instead of adding.

To multiply 2 by 3 is to take 2 three times.

To multiply 4 by 3 is to take 4 three times. 3 times 4 is 12.

To multiply 5 by 2 is to take 5 two times. 2 times 5 is 10.

The sign of multiplication is ×. When placed between two numbers, it means that one is to be taken as many times as there are units in the other. There are 3 units in 3, and there are 4 units in 4. 3 × 4 means that 4 is to be taken 3 times or that 3 is to be taken 4 times. It may be read 3 times 4 or 4 times 3, or 3 multiplied by 4.

TABLE OF 2's

31. *Learn:*

$2\times 1=2$
$2\times 2=4$
$2\times 3=6$
$2\times 4=8$
$2\times 5=10$
$2\times 6=12$
$2\times 7=14$
$2\times 8=16$
$2\times 9=18$
$2\times 10=20$
$2\times 11=22$
$2\times 12=24$

TO THE TEACHER. Multiplication of whole numbers is the process of taking a *number* a certain number of times. It is never a process of taking things. It is a short process of adding a *number* to itself. Pupils have already learned to add numbers.

Have pupils study this table until each separate fact in it can be given instantly, either factor being given first; as, $2\times 3=6$ or $3\times 2=6$.

These facts must be known independently of each other. The pupil who cannot tell how many 2×7 is without going over the table from the beginning to that point does not know the product of 2 times 7.

Close study, many repetitions by the pupil, and quick, sharp drill by the teacher are necessary for the mastery of the Multiplication Table.

1. There are six apples in a basket and they are to be divided equally between two boys.

If each boy takes one apple from the basket, how many are left in the basket?

If each boy takes another apple from the basket, how many are left in the basket?

If each boy takes another apple from the basket, how many are left in the basket? How many apples has each boy?

If they had known how many times 2 can be taken from 6, or how many times 2 is contained in 6, they would have known how many apples to take at first. 3 × 2 is 6; 2 can be taken from 6, 3 times, or 2 is contained in 6, 3 times, or $6 \div 2$ (read 6 divided by 2) equals 3, or $\frac{1}{2}$ of 6 (read one half of 6) equals 3.

The **sign of division** is ÷. When placed between two numbers, it means that the number at its left is to be divided by the number at its right.

To find $\frac{1}{2}$ of a number divide it by 2.

2. If the six apples are to be divided equally among 3 boys and each boy takes 1 apple from the basket, how many apples are left in the basket? If each boy takes another apple from the basket, how many apples are left in the basket? How many apples has each boy?

If they had known how many times 3 can be taken from 6, or how many times 3 is contained in 6, they would have known how many apples to take at first. 2 × 3 is 6. 3 can be taken from 6, 2 times, or 3 is contained in 6, 2 times, or $6 \div 3 = 2$, or $\frac{1}{3}$ of $6 = 2$.

If one number is multiplied by another, the result is called the **product**.

$2 \times 3 = 6$.

6 is the product of 2 times 3 and also of 3 times 2.

If one number is divided by another, the result is called the **quotient**.

If 6 is divided by 3, the quotient is 2.

If 6 is divided by 2, the quotient is 3.

How do you find $\frac{1}{2}$ of a number; $\frac{1}{3}$ of a number; $\frac{1}{4}$ of a number; $\frac{1}{5}$ of a number; $\frac{1}{6}$ of a number; $\frac{1}{7}$ of a number; $\frac{1}{8}$ of a number; $\frac{1}{9}$ of a number; $\frac{1}{10}$ of a number; $\frac{1}{11}$ of a number; $\frac{1}{12}$ of a number?

$2 \times 3 = 6$. $6 \div 2 = 3$. $6 \div 3 = 2$. $\frac{1}{2}$ of $6 = 3$. $\frac{1}{3}$ of $6 = 2$.

Notice that in the five statements only three different results are given, 6, 2, and 3. If the pupil knows any one of these five statements, he ought to be able to give the others.

When the multiplication table is being learned, the division table should be learned, thus:

$3 \times 4 = 12$ and $4 \times 3 = 12$

$12 \div 4 = 3$ and $12 \div 3 = 4$

$\frac{1}{4}$ of $12 = 3$ and $\frac{1}{3}$ of $12 = 4$

To the Teacher. In the drill table below, have pupils read from the book what is given, stating the result in each case. Thus, $2 \times 2 =$ is read $2 \times 2 = 4$. Place the table on the board and point rapidly to the different numbers, requiring pupils to give the indicated result.

32. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $\mathbf{2 \times 2 = 4}$
$\frac{1}{2}$ of $4 = ?$ $4 \div 2 = ?$

2. $\mathbf{2 \times 3 = 6}$
$\frac{1}{2}$ of $6 = ?$ $6 \div 2 = ?$ $6 \div 3 = ?$ $\frac{1}{3}$ of $6 = ?$

3. $\mathbf{2 \times 4 = 8}$
$\frac{1}{2}$ of $8 = ?$ $8 \div 2 = ?$ $8 \div 4 = ?$ $\frac{1}{4}$ of $8 = ?$

4. $\mathbf{2 \times 5 = 10}$
$\frac{1}{2}$ of $10 = ?$ $10 \div 2 = ?$ $10 \div 5 = ?$ $\frac{1}{5}$ of $10 = ?$

5. $\mathbf{2 \times 6 = 12}$
$\frac{1}{2}$ of $12 = ?$ $12 \div 2 = ?$ $12 \div 6 = ?$ $\frac{1}{6}$ of $12 = ?$

6. $\mathbf{2 \times 7 = 14}$
$\frac{1}{2}$ of $14 = ?$ $14 \div 2 = ?$ $14 \div 7 = ?$ $\frac{1}{7}$ of $14 = ?$

7. $\mathbf{2 \times 8 = 16}$
$\frac{1}{2}$ of $16 = ?$ $16 \div 2 = ?$ $16 \div 8 = ?$ $\frac{1}{8}$ of $16 = ?$

8. $\mathbf{2 \times 9 = 18}$
$\frac{1}{2}$ of $18 = ?$ $18 \div 2 = ?$ $18 \div 9 = ?$ $\frac{1}{9}$ of $18 = ?$

9. $\mathbf{2 \times 10 = 20}$
$\frac{1}{2}$ of $20 = ?$ $20 \div 2 = ?$ $20 \div 10 = ?$ $\frac{1}{10}$ of $20 = ?$

10. $\mathbf{2 \times 11 = 22}$
$\frac{1}{2}$ of $22 = ?$ $22 \div 2 = ?$ $22 \div 11 = ?$ $\frac{1}{11}$ of $22 = ?$

11. $\mathbf{2 \times 12 = 24}$
$\frac{1}{2}$ of $24 = ?$ $24 \div 2 = ?$ $24 \div 12 = ?$ $\frac{1}{12}$ of $24 = ?$

12. Give the products of 2 in order, to 12 times 2; thus, 2, 4, 6, 8, etc. Give them in reverse order.

13. Give the number of 2's in each of the following numbers: 4, 10, 18, 6, 12, 24, 20, 16, 8, 14, 22.

33. Drill Table — Different Forms of Questions

To the Teacher. **Have pupils read from the book, giving required products and quotients. Ask the questions and have pupils answer. Have pupils ask the questions and other pupils answer.**

1. Beginning with 2, add by 2's to 24.

2. 2 taken 4 times = ? 8 = how many 2's?

3. 8 taken 2 times = ? 16 = how many 2's?

4. 2 taken 3 times = ? 24 = how many 12's?

5. 6 taken 2 times = ? 24 = how many 2's?

6. 12 taken 2 times = ? 14 = how many 2's?

7. 4 taken 2 times = ? 10 = how many 2's?

8. 2 taken 11 times = ? 14 = how many 7's?

9. 9 taken 2 times = ? 12 = how many 6's?

10. 2 taken 10 times = ? 18 = how many 2's?

11. 7 taken 2 times = ? 22 = how many 2's?

12. 2 taken 5 times = ? 22 = how many 11's?

13. 11 taken 2 times = ? 18 = how many 9's?

14. 9 can be taken from 18 how many times?

15. 2 can be taken from 24 how many times?

16. $18 \div 2 = ?$ $20 \div 10 = ?$ $\frac{1}{7}$ of $14 = ?$

17. $2 \times 8 = ?$ $24 \div 2 = ?$ $\frac{1}{2}$ of $24 = ?$

18. 20 ÷ 2 = ?   16 ÷ 8 = ? $\frac{1}{2}$ of 20 = ?
19. 12 × 2 = ?   22 ÷ 2 = ? $\frac{1}{11}$ of 22 = ?
20. 8 contains 2 how many times? $\frac{1}{6}$ of 12 = ?
21. 14 contains 7 how many times? $\frac{1}{2}$ of 14 = ?
22. 6 contains 2 how many times? $\frac{1}{3}$ of 6 = ?

PROBLEMS AND DRILLS

34. To the Teacher. In class work on problems, several plans may be employed. A pupil may read the problem and give the answer without explanation or analysis; or, after reading the problem, he may call on some other pupil to give the answer. The teacher may read the problem and call upon the pupil to give the answer without explanation or analysis.

A considerable portion of the work in each recitation should be of this class, as it requires rapid work when properly conducted, and gives an opportunity for much practice in performing the various operations.

Another plan is to have the pupil give an analysis of the problem after reading it or hearing it read. The analysis may be in the form of a statement of each operation in order, in such a manner as to show the reason for it, or it may be in the form of question and answer, the pupil stating the first question which must be answered in order to solve the problem, and then answering it. This is followed by the next question and answer, and so on, until the required answer is reached.

When this form of analysis is used, each question should give all the conditions necessary for the correct answer without reference to preceding questions.

Training in the analysis of arithmetical problems should be begun early and continued throughout the course, as it cultivates clear and exact expression and clear thinking. It is not necessary to require a formal analysis of all problems, but there should be enough of this work to satisfy the teacher that each pupil understands what he is doing and to give him the proper training in telling concisely and accurately what he does and why he does it.

Oral

1. How many pints are there in 3 quarts?

Analysis: In 1 quart there are two pints; in 3 quarts there are 3 times 2 pints, or —— pints.

2. —— feet = 1 yard. How many feet are there in 2 yards? Fill the blank.

3. How many pecks = 2 bushels?

4. A pole is 6 feet in length. How many yards long is it?

ANALYSIS: A pole 6 feet long is as many yards long as 3 is contained times in 6, or —— yards long.

5. I buy 4 yards of ribbon at 2 cents a yard. What is the cost?

I give the clerk a dime in payment. How much change do I receive?

6. A gallon contains —— quarts. How many gallons equal 8 quarts?

ANALYSIS: Since there are 4 quarts in 1 gallon, there are as many gallons in 8 quarts as 4 is contained times in 8, or 2 gallons.

7. At 6 cents a quart, what is the cost of a pint of milk?

ANALYSIS: There are 2 pints in 1 quart. If milk costs 6 cents a quart, 1 pint will cost $\frac{1}{2}$ of 6 cents, or —— cents.

8. At 2 cents a foot, what will be the cost of 1 yard of rubber cord?

ANALYSIS: There are 3 feet in 1 yard. At 2 cents a foot 1 yard of rubber cord will cost 3 times 2 cents, or —— cents.

9. How much will 2 yards cost?

10. Henry picks 2 bushels of nuts and sells $\frac{1}{4}$ of them. How many quarts does he sell?

Analysis by Question and Answer: 1st question: How many pecks = 2 bushels? Answer: 8 pecks. 2d question: What is $\frac{1}{4}$ of 8 pecks? Answer: 2 pecks. 3d question: How many quarts = 2 pecks? Answer: 16 quarts.

11. If he sells $\frac{1}{2}$ of them, how many quarts does he sell?

12. John has 6 marbles and gives $\frac{1}{3}$ of them away. How many marbles has he left?

Analysis by Question and Answer: 1st question: If John has 6 marbles and gives $\frac{1}{3}$ of them away, how many does he give away? Answer: 2 marbles. 2d question: If John has 6 marbles and gives away 2, how many has he left? Answer: 4 marbles.

13. If he gives $\frac{2}{3}$ of them away, how many are left?

14. If Mr. Smith's horses eat 2 pecks of oats in a day, in how many days will they eat 2 bushels of oats?

Analysis by Question and Answer: 1st question: How many pecks equal two bushels? Answer: 8 pecks. 2d question: If Mr. Smith's horses eat 2 pecks of oats in 1 day, in how many days will they eat 8 pecks? Answer: 4 days.

15. Jane buys 8 yards of ribbon, and uses $\frac{1}{4}$ of it for her doll. How many yards are left?

16. Make a problem using $\frac{1}{2}$ of any number. Solve it.

17. $3 \times 2 = ?$
$4 \times 2 = ?$
$2 \times 3 = ?$

18. $8 \div 4 = ?$
$6 \div 3 = ?$
$8 \div 2 = ?$

19. $2 \times 4 = ?$
$\frac{1}{2}$ of $4 = ?$
$\frac{1}{3}$ of $6 = ?$

20. $\frac{1}{2}$ of $4 = ?$
$\frac{1}{2}$ of $2 = ?$
$2 \div 2 = ?$

21. $\frac{1}{2}$ of $6 = ?$
$\frac{1}{4}$ of $8 = ?$
$6 \div 2 = ?$

22. $2 \div 1 = ?$
$? + 7 = 16$
$17 - 8 = ?$

Add:

WRITTEN

23.	**24.**	**25.**	**26.**	**27.**
246	285	568	$ 4.56	$ 15.25
327	463	342	3.25	21.17
154	729	756	7.18	84.96

ORAL

28. 4 boys have each 2 books. How many books have all? If 4 boys have 8 books and each boy has 2 books, what part of the whole number has each?

29. 2 nickels = how many cents?

ANALYSIS: 1 nickel = 5¢.
2 nickels = 2 times 5¢, or ——¢.

$\frac{1}{2}$ of 10¢ = ? Give 2 answers.
$\frac{1}{5}$ of 10¢ = ? $\frac{2}{5}$ of 10¢ = ? $\frac{3}{5}$ of 10¢ = ?

30. If $ 12.00 are divided equally between 2 men, how many dollars will each have?

31. If $ 12.00 are divided equally among 6 men, how much will each have?

32. How much is $\frac{1}{6}$ of $ 12.00? How many dollars are $\frac{2}{6}$ of $ 12.00? How many dollars are $\frac{3}{6}$ of 12.00? How much is $\frac{1}{2}$ of $ 12.00?

33. If I give one man $\frac{2}{6}$ of $ 12.00 and another man $\frac{1}{2}$ of $ 12.00, which will have the more, and how much more?

34. A boy buys 2 writing books at 10¢ apiece. How much does he pay for both? Give two answers.

35. If 2 slates cost 20 cents, how much will each cost?

ANALYSIS: Each will cost $\frac{1}{2}$ of 20 cents, or —— cents.

36. If 20 marbles are divided equally between 2 boys, how many will each have? Analyze.

37. If 20 marbles are divided equally among 10 boys, how many will each have?

What part of the whole will each have?

What part of the whole will 2 boys have? 3 boys?

36. What number must be added to 8 to equal 14?

How many 2's must be added to 8 to equal 14?

37. What number must be subtracted from 20 to leave 12?

How many 2's must be subtracted from 20 to leave 12?

38. If there are 9 trees in each of 2 rows, how many are there in both rows? If $\frac{1}{9}$ of the whole number are apple trees, how many apple trees are there? If $\frac{2}{9}$ of the whole number are peach trees, how many peach trees are there? How many apple trees and peach trees?

39. If I pay 20¢ for 2 pounds of candy, how much do I pay for 1 pound?

40. What are $\frac{3}{10}$ of 20?

41. If I divide 20 apples equally among 10 boys, how many will each have?

What part of the whole number will each have?

42. 2 times 11 are how many?

What is $\frac{1}{2}$ of 22? 11 is contained in 22 how many times?

43. If I divide 22 marbles equally among 11 boys, how many marbles will each have? What part of the whole number will each have? What part of the whole number will 2 boys have? 4 boys? How many will 4 boys have? Notice the difference between "what part" and "how many."

44. 12 is what part of 24? 2 is what part of 24? 2 is in 24 how many times? 24 ÷ 12 = ?

Give sums:

45.	8	7	8	7	5	4	6	5
	5	9	4	6	9	7	6	7
46.	28	37	18	37	25	34	56	85
	5	9	4	6	9	7	6	7
47.	9	4	3	2	59	44	73	42
	8	5	7	9	8	5	7	9

Subtract:

WRITTEN

48.	**49.**	**50.**	**51.**	**52.**	**53.**
847	923	500	$14.27	$35.02	75.09
256	478	289	11.63	19.46	28.63

TO THE TEACHER. Many problems are given involving the use of the facts in the table of 2's, in order to accustom pupils to a great variety of forms of statement. This can best be done when the operations are familiar to pupils.

ORAL

54. 4 2's = ? 5 2's = ? 2 5's = ? $10 \div 2 = ?$ $10 \div 5 = ?$

55. $2 \times 5 = ?$ $5 \times 2 = ?$

56. $10 = 2$ taken how many times?
$10 = 5$ taken how many times?

57. How many 2's in 10? How many 5's in 10? How many times can you take 2 from 10? How many times can you take 5 from 10?

58. 6 2's = ? 2 6's = ? $12 = 2$ taken how many times?
$12 = 6$ taken how many times?
$12 \div 2 = ?$ $12 \div 6 = ?$ $\frac{1}{2}$ of $12 = ?$ $\frac{1}{6}$ of $12 = ?$

59. How many 2's in 12? How many 6's in 12?
How many times can you take 2 from 12?
How many times can you take 6 from 12?

60. If I buy 6 yards of cloth at $2 a yard, what is the cost of the cloth?

61. A mother divides a dozen bananas equally among 6 children. How many bananas does each child get?

62. —— inches = 1 foot.
John has a one-foot ruler, and cuts off $\frac{1}{2}$ of it. How many inches does John cut off? How many inches are left?

63. How many months are there in $\frac{1}{2}$ of a year?

64. How many months are there in $\frac{1}{6}$ of a year?

WRITTEN

	65.	66.	67.	68.	69.
Add:	24	15	64	39	80
	36	24	27	47	71
	45	36	34	58	59
	27	43	89	66	68
	93	82	52	75	90
	64	75	78	84	45
	28	46	64	93	33
	37	87	59	42	44
	42	93	33	81	69

ORAL

70. $2 \times 7 = ?$ $7 \times 2 = ?$ $\frac{1}{2}$ of $14 = ?$ $\frac{1}{7}$ of $14 = ?$

71. $14 = 2$ taken how many times? $14 = 7$ taken how many times?

72. If 8 oranges cost 16 cents, what is the cost of one orange?

73. How many hours will it take Daniel to drive 14 miles, if he drives at the rate of seven miles an hour?

74. James and Henry went to visit their uncle. James stayed 14 days and Henry stayed $\frac{1}{7}$ of that time. How many days did Henry stay?

75. How long will 16 bushels of potatoes last, if 2 bushels are used per week?

76.

$7 \times 2 = ?$
$8 \times 2 = ?$
$2 \times 7 = ?$

77.

$2 \times 6 = ?$
$\frac{1}{2}$ of $16 = ?$
$\frac{1}{2}$ of $14 = ?$

78.

$\frac{1}{7}$ of $14 = ?$
$\frac{1}{8}$ of $16 = ?$
$\frac{1}{2}$ of $10 = ?$

79.

$\frac{1}{4}$ of $8 = ?$
$16 \div 2 = ?$
$12 \div 2 = ?$

80.

$8 \div 2 = ?$
$14 \div 2 = ?$
$? \div 2 = 7$

81.

$? \div 3 = 2$
$? \div 7 = 2$
$12 \div ? = 6$

82.

$9 + 6 = ?$
$9 + 7 = ?$
$16 - ? = 9$

83.

$13 - ? = 4$
$? + 4 = 11$
$? - 7 = 5$

84.

$12 \div 6 = ?$
$8 + 9 = ?$
$15 - 9 = ?$

85. If 2 writing books cost 18 cents, what is the cost of 1 writing book?

86. Mary has 20 cents; she gives $\frac{1}{10}$ of her money to her sister. How many cents does her sister get? How many has Mary left?

87. John rides 9 miles on his bicycle on Thursday, and twice as far the next day. How far does he go the next day? In both days?

88. Henry sold 2 quarts of nuts at 8 cents a quart, and spent 9 cents of what he received for candy. How many cents had he left?

89. If 18 marbles are divided equally among 9 boys, each boy receives how many marbles? How many marbles do 2 boys receive? 6 boys?

90.

$9 \times 2 = ?$

$10 \times 2 = ?$

$2 \times 9 = ?$

91.

$2 \times 10 = ?$

$\frac{1}{2}$ of $20 = ?$

$\frac{1}{10}$ of $20 = ?$

92.

$\frac{1}{2}$ of $16 = ?$

$\frac{1}{2}$ of $14 = ?$

$\frac{1}{2}$ of $18 = ?$

93.

$\frac{1}{9}$ of $18 = ?$

$\frac{1}{8}$ of $16 = ?$

$\frac{1}{5}$ of $10 = ?$

94.

$18 \div 2 = ?$

$14 \div 2 = ?$

$20 \div 10 = ?$

95.

$20 \div 2 = ?$

$18 \div 9 = ?$

$? \div 2 = 6$

96.

$? \div 3 = 2$

$18 \div ? = 9$

$20 \div ? = 2$

97.

$9 + 9 = ?$

$7 + 8 = ?$

$4 + 5 = ?$

98.

$16 \div 8 = ?$

$14 \div 2 = ?$

$12 \div 6 = ?$

99. How many stamps at 2 cents each can you buy for 22 cents?

100. A pail holds 24 pints of milk. How many quarts does it hold?

101. James picks one half bushel of nuts. How many pecks does he pick?

102. How many oranges at 2 cents apiece can I buy for 2 dimes?

103. There are how many desks in a row in your schoolroom? How many in 2 rows?

104. 2 is contained in each of the following numbers how many times and how many over: 5, 7, 12, 13, 8, 9, 11, 10, 15, 16, 17, 20, 19, 18, 24, 25, 23, 21, 22?

105. Multiply each of the following numbers by 2 and name the products: 4, 7, 12, 9, 11, 5, 3, 8, 10, 6, 1, 2.

106.
$11 \times 2 = ?$
$12 \times 2 = ?$
$2 \times 11 = ?$

107.
$2 \times 2 = ?$
$\frac{1}{2}$ of $24 = ?$
$\frac{1}{2}$ of $20 = ?$

108.
$\frac{1}{11}$ of $22 = ?$
$\frac{1}{6}$ of $12 = ?$
$\frac{1}{12}$ of $24 = ?$

109.
$24 \div 2 = ?$
$24 \div 12 = ?$
$22 \div 11 = ?$

110.
$\frac{1}{5}$ of $10 = ?$
$\frac{1}{8}$ of $16 = ?$
$22 \div 2 = ?$

111.
$? \div 12 = 2$
$18 \div ? = 2$
$12 \div 6 = ?$

112.
$18 \div 9 = ?$
$? - 6 = 5$
$? + 8 = 9$

113.
$20 \div 10 = ?$
$18 \div 2 = ?$
$\frac{1}{7}$ of $14 = ?$

114.
$12 \div ? = 6$
$16 \div ? = 2$
$18 \div ? = 9$

REVIEW

35. 1. What is the minuend in subtraction?

2. What is the subtrahend in subtraction?

3. What is the difference in subtraction?

4. When a figure in the minuend is less in value than the figure below it, what must be done in order to subtract? What must be done to the next figure in the subtrahend?

5. What is the result called in addition?

6. What is the result called in multiplication?

7. What is the result called in division?

WRITTEN MULTIPLICATION AND DIVISION

36. No partial product, except the last in any example, greater than 8.

Every division exact.

Learn :

Multiplication is the process of taking one number as many times as there are units in another.

Study Recitation

Multiply:

1.	3	6	8	4	5	7	12	9
	2	2	2	2	2	2	2	2

2.	31		
	× 2	2 × 1 = 2.	Write the 2 underneath.
	62	2 × 3 = 6.	Write the 6 at the left of the 2.

3.	Multiplicand	423	2 × 3 = 6.
	Multiplier	× 2	2 × 2 = 4.
	Product	846	2 × 4 = 8.

Learn :

The **multiplicand** is the number multiplied.

The **multiplier** is the number by which we multiply.

The **product** is the number resulting from the multiplication.

WRITTEN

4.	5.	6.	7.	8.	9.
213	224	124	422	523	604
× 2	× 2	× 2	× 2	× 2	× 2

Name multiplicand, multiplier, and product in each example.

Divide:

10. $\begin{array}{r}2\underline{)8}\\4\end{array}$ 2 in 8, 4 times. Write the 4 underneath.

11. $\begin{array}{r}2\underline{)16}\\8\end{array}$ 2 in 16, 8 times. Write the 8 under the 6.

12. $2\underline{)14}$ $8\underline{)16}$ $6\underline{)12}$ $9\underline{)18}$ $12\underline{)24}$ $11\underline{)22}$

13. $\begin{array}{r}2\underline{)468}\\234\end{array}$
2 in 4, 2 times. Write the 2 under the 4.
2 in 6, 3 times. Write the 3 under the 6.
2 in 8, 4 times. Write the 4 under the 8.

14. $\begin{array}{r}2\underline{)608}\\304\end{array}$
2 in 6, 3 times. Write the 2 under the 6.
2 in 0, 0 times. Write the 0 under the 0.
2 in 8, 4 times. Write the 4 under the 8.

15. Divisor $\begin{array}{r}2\underline{)628}\\314\end{array}$ Dividend / Quotient

Learn:

The **dividend** is the number divided.

The **divisor** is the number by which the dividend is to be divided.

The **quotient** is the number resulting from the division.

16. $2\underline{)628}$

17. $2\underline{)426}$

18. $2\underline{)284}$

19. $2\underline{)422}$

20.

$\begin{array}{r}2\underline{)124}\\62\end{array}$
2 in 12, 6 times. Write the 6 under the 2.
2 in 4, 2 times. Write the 2 under the 4.

21.	**22.**	**23.**	**24.**	**25.**
2)1684	2)1824	2)1484	4)84	5)105
26.	**27.**	**28.**	**29.**	**30.**
8)1688	9)189	7)147	4)480	5)10000

31. There are 48 apples in a basket. If half of them are sold to a woman, how many will she receive?

Find $\frac{1}{2}$ of the number by dividing it by 2.

37. Some partial products greater than 9. Some partial dividends not multiples of the divisor.

WRITTEN

Study Recitation

1. Multiply: $$\begin{array}{r}397\\ \underline{2}\\ 794\end{array}$$

2 times 7 is 14. Write the 4 under the 2.

2 times 9 is 18. Carry or add the 1 in the 14 to 18, making 19. Write the 9 at the left of the 4 and under the 9.

2 times 3 is 6; add the 1 in 19 to 6, making 7. Write the 7 at the left of the 9 and under 3.

2. Multiply: $$\begin{array}{r}568\\ \underline{2}\\ 1136\end{array}$$

2 times 8 is 16. Write the 6 under the 2.

2 times 6 is 12; carry or add 1 to the 12, making 13. Write the 3 under the 6 and carry the 1 to 2 times 5, making 11. Write the 11 at the left of the 3.

3. Multiply: $$\begin{array}{r}607\\ \underline{2}\\ 1214\end{array}$$

2 times $7 = 14$. Write the 4 underneath.

2 times $0 = 0$. 1 added to $0 = 1$. Write the 1 underneath.

2 times $6 = 12$. Write the 12 at the left of the 1.

4.	**5.**	**6.**	**7.**	**8.**	**9.**
357	468	504	927	346	576
2	2	2	2	2	2

10.	**11.**	**12.**	**13.**	**14.**	**15.**
212	122	202	309	507	608
4	6	8	2	2	2

Name multiplier, product, and multiplicand in each example.

To the Teacher. **Make no attempt to explain the process of "carrying" at this stage. Later, lead the pupil to discover the reason himself.**

16. Count by 2's from 2 to 24.

17. 2 is in 5 how many times, and how many over? In 6? In 7? In 9? In 13? In 11? In 17? In 25? In 21? In 23? In 19?

Written

Study Recitation

18. Divide: 2)526
263

2 in 5, 2 times and 1 over. Write the 2 under the 5 and think the 1 before the 2, making 12.

2 in 12, 6 times. Write the 6 under the 2.

2 in 6, 3 times. Write the 3 under the 6.

19. Divide: 2)974
487

2 in 9, 4 times and 1 over.
2 in 17, 8 times and 1 over.
2 in 14, 7 times.

20. Divide: $$\begin{array}{r} 2\underline{)604} \\ 302 \end{array}$$

2 in 6, 3 times.
2 in 0, 0 times.
2 in 4, 2 times.

21. Divide: $$\begin{array}{r} 2\underline{)900} \\ 450 \end{array}$$

2 in 9, 4 times and 1 over.
2 in 10, 5 times.
2 in 0, 0 times.

22. $2\underline{)532}$
23. $2\underline{)148}$
24. $2\underline{)698}$
25. $2\underline{)746}$
26. $2\underline{)1354}$

27. $2\underline{)718}$
28. $2\underline{)350}$
29. $2\underline{)894}$
30. $2\underline{)906}$
31. $2\underline{)408}$

32. $4\underline{)804}$
33. $3\underline{)693}$
34. $5\underline{)786}$
35. $2\underline{)574}$
36. $2\underline{)792}$

37. $2\underline{)1574}$
38. $2\underline{)1200}$
39. $2\underline{)700}$
40. $2\underline{)408}$
41. $2\underline{)500}$

42. $2\underline{)908}$
43. $2\underline{)318}$
44. $2\underline{)728}$
45. $2\underline{)1606}$
46. $2\underline{)800}$

PART II

TABLE OF THREES

38. Beginning with 3 add 3's to 36.

Learn:

3 × 1 = 3
3 × 2 = 6
3 × 3 = 9
3 × 4 = 12
3 × 5 = 15
3 × 6 = 18
3 × 7 = 21
3 × 8 = 24
3 × 9 = 27
3 × 10 = 30
3 × 11 = 33
3 × 12 = 36

39. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1.	9 ÷ 3 = ?	$\frac{1}{3}$ of 9 = ?	$\frac{2}{3}$ of 9 = ?
2.	12 ÷ 3 = ?	$\frac{1}{3}$ of 12 = ?	$\frac{1}{4}$ of 12 = ?
3.	15 ÷ 3 = ?	$\frac{1}{3}$ of 15 = ?	$\frac{1}{5}$ of 15 = ?
4.	18 ÷ 3 = ?	$\frac{1}{3}$ of 18 = ?	$\frac{1}{6}$ of 18 = ?
5.	21 ÷ 3 = ?	$\frac{1}{3}$ of 21 = ?	$\frac{1}{7}$ of 21 = ?
6.	24 ÷ 3 = ?	$\frac{1}{3}$ of 24 = ?	$\frac{1}{8}$ of 24 = ?
7.	27 ÷ 3 = ?	$\frac{1}{3}$ of 27 = ?	$\frac{1}{9}$ of 27 = ?
8.	30 ÷ 3 = ?	$\frac{1}{3}$ of 30 = ?	$\frac{1}{10}$ of 30 = ?
9.	33 ÷ 3 = ?	$\frac{1}{3}$ of 33 = ?	$\frac{1}{11}$ of 33 = ?
10.	36 ÷ 3 = ?	$\frac{1}{3}$ of 36 = ?	$\frac{1}{12}$ of 36 = ?

11. Count by 3's from 3 to 36.

12. Three 3's = ?   9 = how many 3's?

13. Eight 3's = ?   12 = how many 4's?

14. Twelve 3's = ?   15 = how many 3's?

15. Nine 3's = ?   30 = how many 3's?

16. Seven 3's = ?   24 = how many 3's?

17. Eleven 3's = ?   27 = how many 9's?

18. Four 3's = ?   33 = how many 3's?

19. Six 3's = ?   36 = how many 12's?

20. Five 3's = ?   18 = how many 3's?

21. Ten 3's = ?   21 = how many 7's?

22. $18 \div 6 = ?$ $15 \div 3 = ?$ $\frac{2}{3}$ of $27 = ?$

23. $27 \div 3 = ?$ $24 \div 8 = ?$ $\frac{1}{9}$ of $27 = ?$

24. $12 \div 4 = ?$ $36 \div 3 = ?$ $\frac{1}{5}$ of $15 = ?$

25. $30 \div 3 = ?$ $21 \div 7 = ?$ $\frac{1}{6}$ of $18 = ?$

26. $33 \div 11 = ?$ $18 \div 3 = ?$ $\frac{1}{3}$ of $33 = ?$

27. $9 \div 3 = ?$ $24 \div 3 = ?$ $\frac{1}{3}$ of $21 = ?$

28. $27 \div 9 = ?$ $36 \div 12 = ?$ $\frac{1}{7}$ of $21 = ?$

29. $\frac{1}{3}$ of $24 = ?$ $\frac{1}{10}$ of $30 = ?$ $\frac{1}{3}$ of $36 = ?$

30. $\frac{1}{3}$ of $12 = ?$ $\frac{1}{3}$ of $27 = ?$ $\frac{1}{8}$ of $24 = ?$

PROBLEMS

40.

ORAL

1. If a man earns $2 a day for 3 days, how many dollars does he earn?

2. If he earns $8 in 4 days, how many dollars does he earn in 1 day?

3. If a man works a number of days at $2 per day and receives $10, how many days does he work?

4. If from 3 pecks of oats I feed a horse 6 quarts, how many quarts have I left?

ANALYSIS: How many quarts equal 3 pecks? Answer: —— quarts. 6 quarts from 24 quarts leaves how many quarts? Answer: —— quarts.

5. If a girl buys 3 pieces of ribbon 3 yards long, how many feet of ribbon does she buy? Analyze.

6. 1 yard = how many inches? $\frac{1}{12}$ of a yard = how many inches? $\frac{3}{12}$ of a yard = how many inches? $\frac{1}{3}$ of a yard = how many inches? $\frac{2}{3}$ of a yard?

7. How many feet = $\frac{2}{3}$ of a yard? 2 feet = how many inches?

THREE AS MULTIPLIER AND DIVISOR

41. WRITTEN

Multiply:

1.	2.	3.	4.	5.	6.	7.	8.
62	73	81	92	63	803	5231	4304
3	3	3	3	3	3	3	3

Prove that your answers are correct by dividing each answer by 3.

9. How much will 33 cords of wood cost at $3 a cord? Prove.

10. A man travels 43 miles an hour by rail. How many miles will he travel in 3 hours? Prove.

11. If a man travels 39 miles in a certain number of hours, how far will he travel in $\frac{1}{3}$ of that time? How do you find $\frac{1}{3}$ of a number?

To THE TEACHER. Use 10, 7, and 12 in place of a "certain number," and lead pupils to see that the number of hours makes no difference in the result.

42. DRILL

To THE TEACHER. This drill in dividing by 3, numbers between 3 and 36 that are not multiples of 3, is a preparation for dividing larger numbers by 3. The greatest difficulty pupils have in short division is in finding the quotient figure when the partial dividend is not exactly divisible by the divisor. If the work below is thoroughly mastered, they will have no trouble in dividing by 3. Similar work should be given with each table.

1. Count by 3's from 3 to 36. Beginning with 36, give in reverse order the numbers exactly divisible by 3: Thus, 36, 33, etc.

2. What is the first number smaller than 8 that contains 3 exactly? The first number smaller than 10 that contains 3 exactly? Than 13? 11? 19? 14? 17? 16? 34? 20? 26? 22? 25? 31? 23? 32? 28? 37? 29? 35?

3. 3 is contained in 7 how many times and how many are left over? 3 in 8? In 10? In 11? In 12? In 15? In 13? In 14?

4. 3 in 18? In 16? In 17? In 21? In 20? In 24?

5. 3 in 22? In 23? In 19? In 27? In 30? In 25?

6. 3 in 28? In 26? In 29? In 35? In 32? In 34?

WRITTEN

Study Recitation

7. Divide: $\begin{array}{r} 3\overline{)1710} \\ 570 \end{array}$

3 in 17, 5 times and 2 over. Place the 5 below the 7. Think the 2 before the 1, making it 21. 3 in 21, 7 times. Place the 7 below the 1. 3 in 0, no times. Place the naught or zero below the zero.

8.	**9.**	**10.**	**11.**	**12.**	**13.**
$3\overline{)87}$	$3\overline{)1242}$	$3\overline{)657}$	$3\overline{)720}$	$2\overline{)346}$	$2\overline{)752}$

Prove that the quotient is correct in each case by multiplying it by the divisor.

14. There are 135 pupils in 3 rooms. If $\frac{1}{3}$ of them are in each room, how many are there in each room?

15. A man has 240 acres in his farm. On $\frac{1}{3}$ of it he raises wheat. How many acres of wheat has he? On $\frac{1}{2}$ of it he raises corn. How many acres of corn has he? On the remainder he raises oats. How many acres of oats does he raise?

Multiply:

16.	**17.**	**18.**	**19.**	**20.**	**21.**
235 3	446 3	255 3	864 3	752 3	234 3

22.	**23.**	**24.**	**25.**	**26.**	**27.**
476 3	879 3	486 3	647 3	784 3	697 3

TABLE OF FOURS

43. Beginning with 4 add 4's to 48.

Learn :

4 × 1 = 4
4 × 2 = 8
4 × 3 = 12
4 × 4 = 16
4 × 5 = 20
4 × 6 = 24
4 × 7 = 28
4 × 8 = 32
4 × 9 = 36
4 × 10 = 40
4 × 11 = 44
4 × 12 = 48

44. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. 16 ÷ 4 = ? $\frac{1}{4}$ of 16 = ? $\frac{3}{4}$ of 16 = ?
2. 20 ÷ 4 = ? $\frac{1}{4}$ of 20 = ? $\frac{1}{5}$ of 20 = ?
3. 24 ÷ 4 = ? $\frac{1}{4}$ of 24 = ? $\frac{1}{6}$ of 24 = ?

TO THE TEACHER. Give similar drill on remaining multiples of 4, to 48.

4. Count by 4's from 4 to 48.
5. Eight 4's = ?   16 = how many 4's ?
6. Twelve 4's = ?   20 = how many 4's ?
7. Nine 4's = ?   24 = how many 6's ?

Continue similar drill.

8. 24 ÷ 6 = ?   20 ÷ 4 = ? $\frac{1}{4}$ of 16 = ?
9. 36 ÷ 9 = ?   28 ÷ 4 = ? $\frac{1}{9}$ of 36 = ?
10. 32 ÷ 4 = ?   48 ÷ 4 = ? $\frac{1}{4}$ of 40 = ?
11. 44 ÷ 11 = ?   40 ÷ 4 = ? $\frac{1}{4}$ of 28 = ?
12. 20 ÷ 5 = ?   48 ÷ 12 = ? $\frac{1}{10}$ of 40 = ?

13. $12 \div 4 = ?$ $16 \div 4 = ?$ $\frac{1}{4}$ of $32 = ?$

14. $36 \div 4 = ?$ $44 \div 11 = ?$ $\frac{1}{7}$ of $28 = ?$

15. $32 \div 8 = ?$ $28 \div 7 = ?$ $\frac{1}{11}$ of $44 = ?$

16. Give in order all the products of 4 to 48.

PROBLEMS

45.

ORAL

1. A earns $ 2 a week; B earns 4 times as many dollars. How many dollars does B earn? If C earns $\frac{1}{2}$ as many dollars as B, how many dollars does C earn? How much do A, B, and C earn in a week? If D earns $\frac{1}{2}$ as many dollars as they all earn in a week, how many dollars does he earn?

2. A man sells 10 cords of wood and receives in payment 4 five-dollar bills. How much does he receive? How much does he receive for 1 cord? For 3 cords?

3. If I divide $ 44 equally among some men, giving each $ 4, how many men are there? How much is $\frac{1}{4}$ of $ 44? How many times $ 4 is $ 44?

4. How many times can I take 4 bushels from 36 bushels?

5. How many times can I fill a gallon measure from a vessel holding 32 quarts?

6. How many yards equal 36 feet?

7. Make a problem using the numbers 4 and 48 and the words *dollars* and *men*. Solve it.

8. I have a stick 9 feet long. How many times must I lay it off on the ground to measure 36 feet? 27 feet? 18 feet?

9. 2 is what part of 12? 4 is what part of 12? 8 is what part of 12?

10. 4 is what part of 16? 32 is how many times 8?

11. What part of 28 is 7? 33 is how many times 11? 24 is how many 8's?

FOUR AS MULTIPLIER AND DIVISOR

46. WRITTEN

Multiply and prove:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
232	156	632	584	327	442	561
4	4	4	4	4	4	4

ORAL

8. Count by 4's from 4 to 48.

See Note to Teacher, p. 110.

9. 4 is contained in the following numbers how many times, and how many over? 12, 16, 15, 13, 14, 20, 24, 18, 19, 17, 23, 21?

10. 28, 35, 27, 36, 32, 34, 33, 48, 44, 45, 47, 42, 39, 29?

WRITTEN

Divide and prove:

11.	**12.**	**13.**	**14.**	**15.**	**16.**
4)484	4)1288	4)1648	4)1764	4)2580	4)3728

17. A farmer has 248 sheep. His son has $\frac{1}{4}$ as many. How many has his son? His brother has 3 times as many as the son. How many has the brother? How many have all three?

18. If a pile of wood contains 136 cords, how many cords will there be in $\frac{1}{4}$ of the pile? In $\frac{1}{2}$ of the pile? In $\frac{1}{3}$ and $\frac{1}{2}$ of the pile?

19. A woman pays $ 7.12 for a dress skirt; $\frac{1}{4}$ as much for a pair of gloves; $\frac{1}{2}$ as much for a pair of shoes. How much does she pay for all? How much more does she pay for the skirt than for the gloves?

20. On a shelf in a store there are 16 rows of cracker boxes, 4 boxes in each row. How many are there in all?

21. I have 30 pencils in one box, 10 in another, and 8 in another, which are to be divided equally among 4 boys. How many pencils will each boy receive?

22. John had 50 cents. After spending 14 cents for himself, he divided the remainder equally among his 4 sisters. How many cents did each sister get?

Divide and prove:

23. $4\overline{)968}$
24. $4\overline{)724}$
25. $4\overline{)572}$
26. $4\overline{)692}$
27. $4\overline{)780}$
28. $4\overline{)3456}$

29. $4\overline{)528}$
30. $4\overline{)760}$
31. $4\overline{)560}$
32. $4\overline{)928}$
33. $4\overline{)368}$
34. $4\overline{)6488}$

TABLE OF FIVES

47. Beginning with 5, add 5's to 60.

Learn:

$\mathbf{5 \times 1 = 5}$ $\qquad$ $\mathbf{5 \times 7 = 35}$

$\mathbf{5 \times 2 = 10}$ $\qquad$ $\mathbf{5 \times 8 = 40}$

$\mathbf{5 \times 3 = 15}$ $\qquad$ $\mathbf{5 \times 9 = 45}$

$\mathbf{5 \times 4 = 20}$ $\qquad$ $\mathbf{5 \times 10 = 50}$

$\mathbf{5 \times 5 = 25}$ $\qquad$ $\mathbf{5 \times 11 = 55}$

$\mathbf{5 \times 6 = 30}$ $\qquad$ $\mathbf{5 \times 12 = 60}$

48. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $25 \div 5 = ?$ $\quad$ $\frac{1}{5}$ of $25 = ?$ $\quad$ $\frac{2}{5}$ of $25 = ?$
2. $30 \div 5 = ?$ $\quad$ $\frac{1}{5}$ of $30 = ?$ $\quad$ $\frac{1}{6}$ of $30 = ?$
3. $35 \div 5 = ?$ $\quad$ $\frac{1}{5}$ of $35 = ?$ $\quad$ $\frac{1}{7}$ of $35 = ?$

To the Teacher. Continue the drill with other multiples of 5, to 60.

4. Subtract by 5's from 60.
5. Eight 5's = ? $\quad$ In 30 how many 5's ?
6. Twelve 5's = ? $\quad$ In 35 how many 5's ?
7. Nine 5's = ? $\quad$ In 60 how many 5's ?

Continue the drill.

8. $\frac{1}{5}$ of $50 = ?$ $\quad$ $\frac{1}{9}$ of $45 = ?$ $\quad$ $\frac{1}{5}$ of $60 = ?$
9. $\frac{1}{6}$ of $30 = ?$ $\quad$ $\frac{1}{5}$ of $45 = ?$ $\quad$ $\frac{1}{8}$ of $40 = ?$
10. $\frac{1}{11}$ of $55 = ?$ $\quad$ $\frac{1}{7}$ of $35 = ?$ $\quad$ $\frac{1}{10}$ of $50 = ?$
11. $\frac{1}{4}$ of $28 = ?$ $\quad$ $55 \div 5 = ?$ $\quad$ $36 \div 4 = ?$
12. $\frac{1}{12}$ of $48 = ?$ $\quad$ $36 \div 3 = ?$ $\quad$ $24 \div 12 = ?$

49. ORAL

1. Five nickels equal how many cents?

2. Five dimes equal what part of a dollar?

3. In five weeks how many days are there?

4. A boy has a kite string 45 yards long. If one ninth of the string is broken off, how many feet are broken off? How long is the part that is left?

5. A flower bed is 5 feet wide. If it is 5 times as long as it is wide, what is its length?

6. If a dozen pencils cost 60 ¢, what is the cost of 1 pencil? Of 3 pencils?

7. How many nickels equal a quarter of a dollar? How many quarters of a dollar equal a half dollar?

How many nickels equal a half dollar?

EXAMPLES AND PROBLEMS

50. ORAL

See Note to Teacher, p. 110.

How many times and how many over, is 5 contained in the following numbers?

1. 20, 21, 24, 25, 35, 37, 39, 40, 45, 42, 44, 60, 54.
2. 58, 55, 57, 59, 48, 38, 29, 19, 43, 33, 46, 27, 41.

WRITTEN

Multiply and prove: Divide and prove:

3.	**4.**	**5.**	**6.**	**7.**	**8.**
25 5	126 5	847 5	5)5280	5)6375	5)4220

9. A man divides $20 ($5800) among his wife and four children, giving each the same amount. How many dollars will each have? First solve the problem using $20; then use the number in the parenthesis.

10. A man earns $70 a month and spends $52 a month. How much does he save in three months? What is the first question to be answered?

11. A man travels 96 miles in three days. At the same rate how far will he travel in five days? What is the first question to be answered?

12. If it costs $2 a square yard to put down asphalt pavement, how much will it cost to pave 15 square yards? 35 square yards? 1824 square yards?

13. A man saves $165, saving $5 a week. How many weeks does it take him?

14. A piece of cloth is 60 yards long. $\frac{1}{12}$ of it is sold for $3 a yard, and $\frac{1}{5}$ of it for $2 a yard. How much is received for each part? For both parts?

15. A man has twelve horses. He keeps seven of them and sells the remainder at $75 a head. How much does he get for them?

Add:

16.	**17.**	**18.**	**19.**	**20.**	**21.**
4728	27	273	189	643	169
1365	49	146	973	752	478
4237	38	352	468	856	986
3856	64	437	572	479	748

TABLE OF SIXES

51. Add by 6's from 6 to 72.

Learn:

$6 \times 1 = 6$
$6 \times 2 = 12$
$6 \times 3 = 18$
$6 \times 4 = 24$
$6 \times 5 = 30$
$6 \times 6 = 36$
$6 \times 7 = 42$
$6 \times 8 = 48$
$6 \times 9 = 54$
$6 \times 10 = 60$
$6 \times 11 = 66$
$6 \times 12 = 72$

52. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $36 \div 6 = ?$ $\frac{1}{6}$ of $36 = ?$ $\frac{2}{6}$ of $36 = ?$
2. $42 \div 6 = ?$ $\frac{1}{6}$ of $42 = ?$ $\frac{1}{7}$ of $42 = ?$
3. $48 \div 6 = ?$ $\frac{1}{6}$ of $48 = ?$ $\frac{1}{8}$ of $48 = ?$

Continue the drill.

4. Count by 6's from 6 to 72.
5. Eight 6's = ? In 48 how many 6's?
6. Twelve 6's = ? In 72 how many 6's?
7. Nine 6's = ? In 60 how many 6's?

Continue the drill.

8. $\frac{1}{8}$ of $48 = ?$ $\frac{1}{6}$ of $72 = ?$ $\frac{1}{10}$ of $60 = ?$
9. $6 \times 7 = ?$ $11 \times 6 = ?$ $9 \times 6 = ?$
10. $6 \times 6 = ?$ $\frac{1}{6}$ of $54 = ?$ $\frac{1}{6}$ of $48 = ?$
11. $\frac{1}{6}$ of $36 = ?$ $6 \times 12 = ?$ $\frac{1}{5}$ of $30 = ?$

12. 8 can be taken from 48 how many times? $\frac{1}{8}$ of 48 = ?

13. 11 can be taken from 66 how many times? $\frac{1}{6}$ of 36 = ?

14. 6 can be taken from 72 how many times? $\frac{1}{10}$ of 60 = ?

15. 6 can be taken from 36 how many times? $\frac{1}{6}$ of 54 = ?

16. 6 can be taken from 42 how many times? $\frac{1}{6}$ of 42 = ?

17. Beginning with 1, add 6's to 73.

18. Beginning with 3, add 6's to 75.

19. Beginning with 4, add 6's to 52.

20. Beginning with 8, add 6's to 62.

EXAMPLES AND PROBLEMS

53. ORAL

1. Give the products of the following numbers by 6: 5, 2, 7, 9, 12, 8, 11, 9, 10.

2. Give one sixth of the following numbers: 24, 18, 12, 30, 42, 54, 48, 60, 72, 66, 36.

3. If you write 4 rows of numbers, 6 numbers in each row, how many numbers do you write in all?

4. If 6 hills of corn are planted 4 feet apart in a row, how many feet are there between the end hills? Show by diagram on the board.

5. John has six times as much money as James, who has seven cents. How much more has John than James? The first question to be answered is what?

6. From three o'clock until nine o'clock is what part of the day? How many hours are there in a day? How many hours is it from 3 o'clock until 9 o'clock?

7. At 6 cents a quart, what will be the cost of 2 gallons of milk? Of 3 gallons? Of 1 gallon? Of $\frac{1}{2}$ gallon?

8. A boy borrows 60 cents from his brother, and pays it to him at 12 cents a week. How many weeks does it take him to pay it? If he had paid it at the rate of 6 cents a week, how many weeks would it have taken?

9. A farmer sells 6 calves at $7 apiece, and 5 sheep at $2 apiece. How much does he receive for the calves? How much does he receive for the sheep? How much does he receive for the sheep and calves?

10. If bananas sell at 12 cents a dozen, how many can I buy for 60 cents? How much will a half dozen cost? How many half dozen can I buy for 72 cents? How many dozen?

11. A man employs 6 clerks at $5 a week and 3 clerks at $10 a week. How much does he pay them all per week?

12. A room is 10 yards long and 6 yards wide. What is the distance around it in yards?

13. Give $\frac{1}{12}$ of the following numbers: 24, 48, 36, 60, 72. Give $\frac{3}{12}$ of the same numbers; $\frac{5}{12}$; $\frac{7}{12}$; $\frac{9}{12}$.

14. Add by 6's from 6 to 72.

15. Subtract by 8's from 48. Add by 8's from 8 to 48.

If a number will not exactly divide another, what is left may be written as follows:

16. $$\begin{array}{r} 6)\underline{49} \\ 8\frac{1}{6} \end{array}$$

6 in 49 eight times and 1 over. Write 1 over the 6 at the right of the 8 and read the answer $8\frac{1}{6}$ (eight and one sixth).

17. $$\begin{array}{r} 12)\underline{38} \\ 3\frac{2}{12} \end{array}$$

12 in 38 three times and 2 over. Write it $3\frac{2}{12}$. Read the answer (three and two twelfths).

18. Subtract by 6's from 72.

See Note to Teacher, p. 110.

19. Divide each of the following numbers by 6: 36, 38, 39, 45, 47, 50, 54, 56, 60, 65, 68, 71, 25, 19, 17, 13.

20. Add by 9's to 54.

21. Divide each of the following numbers by 9: 19, 28, 37, 47, 56, 58, 17, 13, 22.

22. Add by 8's from 8 to 48.

23. Divide each of the following numbers by 8: 40, 44, 32, 36, 34, 27, 29, 35, 43, 38.

WRITTEN

Divide the following numbers by 6 and prove:

24. 216 726 594 678 234 2412

Divide each of the following numbers by 9:

25. 288 468 5634 3025 4612 3894

Divide each of the following numbers by 6:

26. 312 540 7266 3024 3654 4330

27. If a man pays 28 cents for each of 6 chickens, how much does he pay for all?

28. A man earns 26 dollars a month. How much will he earn in 9 months? How much more would he earn in a year than in 9 months?

29. If a man's salary is $ 287.55 a year, how much will it be for 6 years?

30. A salesman sells three bills of goods. The first amounts to $ 245; the second to three times as much; and the third to as much as the other two. What is the amount of the three bills?

Teacher explain the meaning of *bill.* Illustrate.

31. Multiply 248 by 6 and divide the product by 5. Which number is the multiplicand? Which is the multiplier? Which is the product? Which is the divisor? Which is the dividend? Which is the remainder? (The **remainder** is what is left over when the division is not exact.)

MULTIPLICATION WITH MULTIPLIERS OF TWO FIGURES

54.

WRITTEN

Study Recitation

1. Multiply:

$$\begin{array}{r} 267 \\ 23 \\ \hline 801 \\ 534 \\ \hline 6141 \end{array}$$

Multiply by 3; the product is 801.

Multiply by 2; 2 times 7 is 14; place the 4 under the 0 and the 2. Add the 1 to 2 times 6, making 13. Place the 3 at the left of the 4 under the 8. Add the 1 to 2 times 2, making 5. Place the 5 at the left of the 3.

Each of the two products is called a **partial product**. Add the two partial products. The sum is 6141.

Multiply:

2.	3.	4.	5.	6.	7.
256	327	234	505	729	384
24	32	26	16	25	46

PROBLEMS

55.

ORAL AND WRITTEN

1. If a furniture dealer buys 6 chairs at $ 2.75 each and 4 tables at $9 each, how much is his bill?

2. How much will 24 cups cost at 6 cents each?

3. If a carpet costs $ 3 ($ 2.25) a yard, how much will 10 (28) yards cost?

NOTE. When numbers occur in *parenthesis* in a problem, like ($ 2.25) and (28) in this problem, first solve the problem, using the numbers without parenthesis, and then solve, using the numbers in parenthesis. The problem to be solved first is: If a carpet costs $ 3 a yard, how much will 10 yards cost? The second problem is: If a carpet costs $2.25 a yard, how much will 28 yards cost?

4. In a fourth grade there are 36 pupils. If each buys an arithmetic at 35 cents, how much do they all pay for their arithmetics? If each buys a speller at 20 cents, how much do the spellers cost? If they pay 45 cents each for readers, how much do all pay for readers? How much does the entire grade pay for the three books?

5. There are 6 kegs of nails each weighing 125 pounds; and the kegs that contain the nails weigh 8 pounds. Make two problems from these statements and solve each.

6. A man buys 25 bushels of potatoes at 75 cents a bushel, and gives in exchange 6 sacks of flour at $1.35 per sack. How much does he still owe? In solving this problem make and answer three questions.

7. At the rate of 8 oranges for 24 cents, what will be the cost of 16 oranges? Of two dozen oranges? Of four dozen oranges? Four dozen is how many times 8? What is the first question to be answered?

8. A man saves $512 in eight months. How many dollars does he save in each month? Counting four weeks in a month, how much does he save each week?

9. A merchant's sales in two years amounted to $8948. What were his average sales per year?

10. A square yard of oil cloth covers 9 square feet. How many square yards of oil cloth would it take to cover a floor that contains 117 square feet? Show on the board that a square yard of oil cloth covers 9 square feet.

Teacher explain meaning of *square yard.*

11. A teacher desires to put 35 pupils into four classes. She puts 9 pupils in each of three classes. How many must she put into the fourth class?

12. There are 288 lines on 8 pages. How many lines are there on 1 page? On 6 pages?

13. If there are 250 lines on 5 pages, how many lines are there on 3 pages? What is the first question to be answered? The second?

14. A seamstress earns $ 12 a week and spends $ 4 a week for board and $ 1 a week for other expenses. In how many weeks will she save $ 35? Ask and answer three questions in working this problem.

ORAL

DRILL IN ADDITION

56. Add rapidly to each of the following numbers first 5, then 8, 7, 6, 4, 3, 9, 2:

1. 17, 27, 34, 54, 63, 73, 48, 68, 56, 26, 35, 75, 33.
2. 22, 62, 49, 79, 28, 46, 53, 19, 16, 37, 45, 18, 13.
3. 86, 72, 84, 93, 64, 89, 88, 31, 61, 24, 82, 43, 23.

TABLE OF SEVENS

57. Add by 7's from 7 to 84.

Learn:

$7 \times 1 = 7$
$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
$7 \times 5 = 35$
$7 \times 6 = 42$
$7 \times 7 = 49$
$7 \times 8 = 56$
$7 \times 9 = 63$
$7 \times 10 = 70$
$7 \times 11 = 77$
$7 \times 12 = 84$

58. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $49 \div 7 = ?$ $\frac{1}{7}$ of $49 = ?$ $\frac{2}{7}$ of $49 = ?$
2. $56 \div 7 = ?$ $\frac{1}{8}$ of $56 = ?$ $\frac{1}{7}$ of $56 = ?$

Continue the drill.

3. Count by 7's from 7 to 84.
4. Eight 7's = ? 12 taken 7 times = ?
5. Twelve 7's = ? 8 taken 7 times = ?
6. $42 \div 6 = ?$ $63 \div 7 = ?$ $\frac{1}{11}$ of $77 = ?$
7. $21 \div 7 = ?$ $84 \div 12 = ?$ $\frac{1}{7}$ of $63 = ?$
8. $77 \div 11 = ?$ $42 \div 7 = ?$ $\frac{1}{10}$ of $70 = ?$
9. $35 \div 5 = ?$ $28 \div 7 = ?$ $\frac{1}{12}$ of $84 = ?$
10. $63 \div 9 = ?$ $84 \div 7 = ?$ $\frac{1}{5}$ of $35 = ?$
11. $70 \div 7 = ?$ $35 \div 7 = ?$ $\frac{1}{7}$ of $70 = ?$
12. $56 \div 8 = ?$ $21 \div 3 = ?$ $\frac{1}{9}$ of $63 = ?$
13. $14 \div 7 = ?$ $70 \div 10 = ?$ $\frac{1}{7}$ of $77 = ?$

14. Beginning with 1 add 7's to 85.

15. Beginning with each number from 2 to 7 add 7's to 90.

59. ORAL

1. Give the products of 7 by each of the following numbers: 6, 8, 9, 11, 10, 12, 4, 7, 3, 2.

2. Multiply each of the following numbers by 4: 12, 8, 7, 11, 9, 10, 5.

3. Give $\frac{1}{7}$ of each of the following numbers: 21, 35, 49, 28, 84, 77, 63, 70, 56, 42. Give $\frac{2}{7}$ of each of these numbers: $\frac{5}{7}$, $\frac{6}{7}$, $\frac{3}{7}$, $\frac{4}{7}$.

4. Add by 7's to 84, beginning with 7; with 2; with 5; with 8.

See Note to Teacher, p. 110.

Give results:

5.	**6.**	**7.**	**8.**	**9.**	**10.**
$7\underline{)24}$	$7\underline{)27}$	$7\underline{)37}$	$7\underline{)46}$	$7\underline{)54}$	$7\underline{)34}$
11.	**12.**	**13.**	**14.**	**15.**	**16.**
$7\underline{)64}$	$7\underline{)58}$	$7\underline{)48}$	$7\underline{)38}$	$7\underline{)55}$	$7\underline{)68}$

17. How many days are there in six weeks? In 63 days how many weeks are there? How many months of 4 weeks each?

18. A man earns $56 in 4 weeks. Another man earns $\frac{1}{7}$ as much in 1 week. How much does the second man earn in a week? How much can he earn in 4 weeks?

19. A milliner buys 12 hats at $5 apiece and sells them at $7 apiece. How much does she gain? Solve in two ways. Analyze.

20. A merchant sold overcoats at $12.50 which cost $10.50. How much did he gain on 12 coats? Solve in two ways. Analyze.

21. A bookseller bought 7 books at 12¢ apiece, and sold them at a gain of 3¢ apiece. For how much did he sell them? Solve in two ways. Which is the better way? Why?

22. If a miller sells 8 barrels of flour at $6 a barrel, and buys with the proceeds cloth at $4 a yard, how many yards does he purchase?

23. I pay $35 for coal at $7 a ton. How many tons do I purchase?

24. A man buys 6 car tickets for 25¢. How many can he buy for 50¢? For $1? For $\frac{3}{4}$ of a dollar?

25. Oranges are selling at 3 for 5¢. How many can I buy for 20¢?

ANALYSIS: I can buy as many times 3 oranges as 5 is contained times in 20, or 4 times 3 oranges, which = —— oranges.

26. If oranges had been selling at 4 for 10¢, how many could I have bought for 20¢?

27. James had 12 chickens. He sold $\frac{2}{3}$ of them

How many did he sell? Ask another question on the same statement. Answer the question.

28. If a boy paid $\frac{1}{10}$ of his money for a pencil, $\frac{3}{4}$ of it for an arithmetic, and $\frac{1}{8}$ for a spelling blank, and had 40¢ before he made his purchases, how much had he left? Ask and answer five questions in solving.

29. In an arithmetic lesson there were 30 problems. John solved $\frac{2}{3}$ of them and Henry solved $\frac{9}{10}$ of them. How many did each solve?

30. How much will $\frac{3}{4}$ of a pound of raisins cost at 20¢ a pound? How much will $\frac{1}{2}$ pound cost? Ask and answer another question.

31. 4 gallons of milk filled 8 pitchers of equal size. How much did each pitcher hold?

REVIEW

60. **1.** What is $\frac{1}{8}$ of 48? $\frac{3}{8}$ of 48? $\frac{3}{4}$ of 24? $\frac{2}{3}$ of 15? $\frac{3}{5}$ of 20? $\frac{5}{6}$ of 24? $\frac{3}{8}$ of 32?

2. Find $\frac{2}{3}$ of the following numbers: 9, 12, 18, 15, 21, 24, 30, 27, 36, 33.

3. Find $\frac{3}{4}$ of the following numbers: 12, 8, 16, 24, 32, 20, 36, 48, 44, 40.

4. Find $\frac{3}{7}$ of the following numbers: 14, 7, 28, 35, 49, 21, 56, 70, 63, 84.

5. Find $\frac{3}{8}$ of the following numbers: 56, 24, 16, 32, 48, 40, 8.

6. Find $\frac{3}{12}$ of the following numbers: 24, 48, 36, 84, 60, 72, 12.

7. Find $\frac{3}{5}$ of the following numbers: 10, 15, 30, 20, 45, 25, 60, 35, 55, 40, 50.

8. Give the products of 8, 4, 12, 7, 11, 9, 6, 5, 10, 3, by 3; by 5; by 7; by 4; by 6; by 2.

9. 84, 72, 69, 48, 56, 45, 37, 23, 91.

To each of the numbers in Ex. 9, add 9; 7; 5; 8; 6; 4; 3. Subtract the same numbers.

PROBLEMS

61. WRITTEN

1. If 50 pupils in the first grade rent the books they use in school for 25¢ each a year, how much will the whole grade pay for the use of their books?

2. If the second grade contains 34 pupils and each pays 35¢ a year for the use of books in school, how much will this grade pay each year?

3. If the third grade contains 47 pupils and they pay 45¢ each for the use of school books per year, how much does this grade pay?

4. Using the facts stated in two or more of these problems, make two or more problems and solve them.

5. A farmer sells a grocer 25 pounds of butter at 24¢ a pound, and takes in exchange fruit jars at 12¢ apiece. How many jars does he get? Work in two ways.

6. Two boys, A and B, travel in the same direction, A at the rate of 5 miles per hour and B at the rate of 3 miles per hour. If they start from the same point at the same time, how soon will A be 24 miles ahead of B?

What is the first question to be answered in solving this problem?

How far would each have traveled at the end of 1 hour? Show by diagram on the board. Suppose they travel in opposite directions, how far apart will they be in 1 hour? In 3 hours?

7. A man earns $21 a week; his expenses are $10 a week. In how many weeks can he save $66?

Multiply:

8.	9.	10.	11.
257	456	982	4257
37	64	25	7

12. If eggs are 12¢ a dozen, how many eggs can be bought for $3.84?

13. How many pounds of sugar at 6¢ a pound will it take to pay for the eggs?

14. Make a drawing showing the number of square feet in a square yard.

15. It costs $1.89 per square yard to pave a street. How much will it cost per square foot? Analyze.

TABLE OF EIGHTS

62. Add by 8's from 8 to 96.

Learn :

$\mathbf{8 \times 1 = 8}$ $\qquad$ $\mathbf{8 \times 7 = 56}$

$\mathbf{8 \times 2 = 16}$ $\qquad$ $\mathbf{8 \times 8 = 64}$

$\mathbf{8 \times 3 = 24}$ $\qquad$ $\mathbf{8 \times 9 = 72}$

$\mathbf{8 \times 4 = 32}$ $\qquad$ $\mathbf{8 \times 10 = 80}$

$\mathbf{8 \times 5 = 40}$ $\qquad$ $\mathbf{8 \times 11 = 88}$

$\mathbf{8 \times 6 = 48}$ $\qquad$ $\mathbf{8 \times 12 = 96}$

63. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $64 \div 8 = ?$ $\qquad$ $\frac{1}{8}$ of $64 = ?$ $\qquad$ $\frac{3}{8}$ of $64 = ?$
2. $72 \div 8 = ?$ $\qquad$ $\frac{1}{8}$ of $72 = ?$ $\qquad$ $\frac{1}{9}$ of $72 = ?$

Continue the drill.

3. Count by 8's to 96 beginning with 8.
4. Ten 8's = ? $\qquad$ In 72 how many 8's?
5. Eight 8's = ? $\qquad$ In 96 how many 8's?

Continue the drill.

6. $72 \div 9 = ?$ $\qquad$ $56 \div 7 = ?$ $\qquad$ $\frac{1}{8}$ of $96 = ?$
7. $8 \times 11 = ?$ $\qquad$ $64 \div 8 = ?$ $\qquad$ $\frac{1}{8}$ of $88 = ?$
8. $48 \div 8 = ?$ $\qquad$ $8 \times 8 = ?$ $\qquad$ $\frac{1}{10}$ of $80 = ?$
9. $8 \times 5 = ?$ $\qquad$ $4 \times 8 = ?$ $\qquad$ $\frac{1}{8}$ of $64 = ?$

64. ORAL EXERCISES

1. Give the product of each of the following numbers by 8: 9, 10, 8, 12, 11, 7, 6, 5, 4, 3, 2.

2. Give the quotient of the following numbers by 8: 24, 40, 56, 64, 80, 96, 88, 32, 16, 48, 72.

To the Teacher. See note, p. 110.

3. Give $\frac{1}{8}$ of the following numbers: 18, 26, 34, 43, 60, 69, 73, 71, 85, 87, 89, 92, 96, 90.

4. Give the sum of $\frac{1}{8}$ and $\frac{1}{4}$ of each of the following numbers: 24, 32, 40, 48.

5. Give the sum of $\frac{1}{6}$ and $\frac{1}{3}$ of each of the following numbers: 12, 18, 24, 36, 30.

6. Give the sum of $\frac{1}{2}$ and $\frac{3}{4}$ of each of the following numbers: 4, 8, 12, 20, 16, 24.

7. Give the difference between $\frac{1}{3}$ and $\frac{2}{9}$ of each of the following numbers: 18, 36, 27.

8. A man has 72 bushels of wheat. He sells $\frac{5}{12}$ of it to one man and $\frac{3}{6}$ of it to another man. How many bushels has he left?

9. A man earns $72 in one month; his son earns $\frac{5}{8}$ as many dollars in the same time. Make 2 problems from these statements. Solve each.

Learn: **16 ounces = 1 pound**

To the Teacher. By illustration or otherwise, see that pupils have definite ideas of pound and ounce.

10. How many ounces are there in a half pound? In $1\frac{1}{2}$ pounds?

11. In 1 pound and 12 ounces, how many ounces are there?

12. If an 8-ounce loaf of bread costs 5¢, how much should a pound loaf cost? How much should 4 pound loaves cost?

13. What two numbers multiplied together will give 64? 72? 96? 84? 36?

14. What sets of two numbers multiplied together will give 24? 20? 18? 30? 40? 60? Use no number larger than 12.

15. How much will $\frac{3}{4}$ of a pound of butter cost at 20¢ a pound?

16. How much will $\frac{5}{8}$ of a pound of butter cost at 32¢ a pound?

17. $\frac{1}{4}$ of a pound is how many ounces?

18. 4 ounces is what part of a pound? 2 ounces is what part of a pound? How many 8ths of a pound equal 4 ounces?

19. $\frac{2}{8}$ of a pound = what other part of a pound?

20. Compare $\frac{2}{8}$ of 24 with $\frac{1}{4}$ of 24. $\frac{2}{8}$ of 32 with $\frac{1}{4}$ of 32. $\frac{2}{8}$ of 40 with $\frac{1}{4}$ of 40. $\frac{2}{8}$ of 48 with $\frac{1}{4}$ of 48. $\frac{2}{8}$ of a number = what part of it?

21. How do you find $\frac{1}{8}$ of a number?

22. How may you find $\frac{2}{8}$ of a number without dividing by 8?

23. In $\frac{2}{4}$ of a pound how many ounces are there?

24. What other part of a pound equals the same number of ounces?

25. $\frac{2}{4}$ of 2 dozen are how many? $\frac{1}{2}$ of 2 dozen are how many?

26. $\frac{2}{4}$ of a number = what other part of it?

27. Show it by the use of 8 objects; 12 objects.

Learn: *$\frac{1}{2}$ of a number = $\frac{2}{4}$ of the number*
$\frac{2}{4}$ of a number = $\frac{1}{2}$ of the number

28. John pays 56¢ for a book, and $\frac{1}{8}$ as much for ink. How much does the ink cost? Make two more questions on these statements.

29. How may you find $\frac{1}{4}$ of 56 without dividing it by 4?

To the Teacher. See note, p. 110.

30. Give the quotient of each of the following numbers divided by 8: 48, 72, 96, 88, 80, 56, 59, 65, 73, 70, 85, 90.

31. Give the quotient of each of the following numbers divided by 9; by 10; by 11; by 12: 70, 67, 71, 56, 47, 57, 43, 37, 34, 39, 23.

65.

Written

Divide:

1.	**2.**	**3.**	**4.**	**5.**
$8\overline{)2648}$	$7\overline{)5405}$	$9\overline{)3725}$	$12\overline{)85361}$	$11\overline{)2785}$

6. The meals for a family of 4 persons cost $ 1.75 a day. How much will they cost for the month of January? January has 31 days.

7. If 4 car loads of apples cost $800, how much will 1 car load (7 car loads) cost? If there are 100 barrels in a car, how much will a barrel cost?

8. If 8 head of cattle cost $216.56, how much do 16 head cost? Work in two ways. Which is the shorter?

ORAL

9. How much will 2½ dozen eggs cost at 10¢ a dozen? At 12¢ a dozen?

ANALYSIS: 2 dozen eggs, at 10¢ a dozen, cost 2 × 10¢ = 20¢. ½ dozen eggs cost ½ of 10¢ = 5¢. 2½ dozen eggs cost 20¢ + 5¢ = —— cents.

10. $\frac{1}{5}$ of a number is 8. What is the number? What are $\frac{2}{5}$ of the same number? $\frac{3}{4}$ of the same number?

11. If 24 is $\frac{3}{5}$ of some number, how can you find $\frac{1}{5}$ of that number?

12. If 12 is $\frac{3}{5}$ of some number, what is $\frac{1}{5}$ of that number? How can you find the number?

13. If you have $\frac{3}{7}$ of a number, how can you find the number?

14. $\frac{3}{7}$ of a number is 15. What is the number?

15. John drives 56 miles in a day; Henry drives $\frac{7}{8}$ as far. How many miles does Henry drive?

16. How many are $\frac{7}{8}$ of 56?

17. A bolt of ribbon contains 10 yards. If a girl pays 20¢ for 4 yards, how much will the whole bolt cost at the same rate?

25. $\frac{2}{4}$ of 2 dozen are how many? $\frac{1}{2}$ of 2 dozen are how many?

26. $\frac{2}{4}$ of a number = what other part of it?

27. Show it by the use of 8 objects; 12 objects.

Learn: *$\frac{1}{2}$ of a number = $\frac{2}{4}$ of the number*
$\frac{2}{4}$ of a number = $\frac{1}{2}$ of the number

28. John pays 56¢ for a book, and $\frac{1}{8}$ as much for ink. How much does the ink cost? Make two more questions on these statements.

29. How may you find $\frac{1}{4}$ of 56 without dividing it by 4?

To the Teacher. See note, p. 110.

30. Give the quotient of each of the following numbers divided by 8: 48, 72, 96, 88, 80, 56, 59, 65, 73, 70, 85, 90.

31. Give the quotient of each of the following numbers divided by 9; by 10; by 11; by 12: 70, 67, 71, 56, 47, 57, 43, 37, 34, 39, 23.

65.

WRITTEN

Divide:

1. 8)2648

2. 7)5405

3. 9)3725

4. 12)85361

5. 11)2785

6. The meals for a family of 4 persons cost $1.75 a day. How much will they cost for the month of January? January has 31 days.

7. If 4 car loads of apples cost \$800, how much will 1 car load (7 car loads) cost? If there are 100 barrels in a car, how much will a barrel cost?

8. If 8 head of cattle cost \$216.56, how much do 16 head cost? Work in two ways. Which is the shorter?

ORAL

9. How much will $2\frac{1}{2}$ dozen eggs cost at 10¢ a dozen? At 12¢ a dozen?

ANALYSIS: 2 dozen eggs, at 10¢ a dozen, cost $2 \times 10¢ = 20¢$. $\frac{1}{2}$ dozen eggs cost $\frac{1}{2}$ of $10¢ = 5¢$. $2\frac{1}{2}$ dozen eggs cost $20¢ + 5¢ =$ —— cents.

10. $\frac{1}{5}$ of a number is 8. What is the number? What are $\frac{2}{5}$ of the same number? $\frac{3}{4}$ of the same number?

11. If 24 is $\frac{3}{5}$ of some number, how can you find $\frac{1}{5}$ of that number?

12. If 12 is $\frac{3}{5}$ of some number, what is $\frac{1}{5}$ of that number? How can you find the number?

13. If you have $\frac{3}{7}$ of a number, how can you find the number?

14. $\frac{3}{7}$ of a number is 15. What is the number?

15. John drives 56 miles in a day; Henry drives $\frac{7}{8}$ as far. How many miles does Henry drive?

16. How many are $\frac{7}{8}$ of 56?

17. A bolt of ribbon contains 10 yards. If a girl pays 20¢ for 4 yards, how much will the whole bolt cost at the same rate?

18. If $\frac{4}{10}$ of a bolt of ribbon costs 12¢, how much will the whole bolt cost at the same rate?

19. 10 is $\frac{5}{8}$ of what number? 6 is $\frac{3}{4}$ of what number?

20. If you have $\frac{7}{8}$ of 56, how can you find $\frac{1}{8}$ of it?

WRITTEN

Find correct results:

21. $276 + 138 + 257 - 463 = ?$

22. $785 - 236 + 425 - 222 = ?$

23. $8567 \div 6 = ?$

24. $4273 \div 5 = ?$

ORAL DRILL IN ADDITION

66. To the Teacher. Give pupils drill by requiring them to add each number less than 10 to the numbers in columns, and in each line from left to right, giving results orally. Time them for rapid work.

	a	*b*	*c*	*d*	*e*	*f*
1.	6	16	26	46	56	96
2.	9	19	29	49	59	89
3.	2	12	22	42	52	62
4.	8	18	28	48	58	38
5.	5	15	25	45	55	65
6.	7	17	27	47	57	77
7.	4	14	24	44	54	84
8.	1	11	21	41	51	71
9.	3	13	23	43	53	93

Illustration: Add 9 to each number in column *a*, beginning with 6; thus: 15, 18, 11, 17, etc. In the same way add other numbers less than 10.

Add 8 to each number in the first line; thus: 14, 24, 34, 54, 64, 104. Do similar work with numbers in the other lines.

67. TABLE OF NINES

Add by 9's from 9 to 108.

Learn:

$9 \times 1 = 9$ **$9 \times 7 = 63$**
$9 \times 2 = 18$ **$9 \times 8 = 72$**
$9 \times 3 = 27$ **$9 \times 9 = 81$**
$9 \times 4 = 36$ **$9 \times 10 = 90$**
$9 \times 5 = 45$ **$9 \times 11 = 99$**
$9 \times 6 = 54$ **$9 \times 12 = 108$**

68. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $9 \times 9 = ?$ $81 \div 9 = ?$ $\frac{1}{9}$ of $81 = ?$
2. $10 \times 9 = ?$ $90 \div 9 = ?$ $\frac{1}{9}$ of $90 = ?$
3. $11 \times 9 = ?$ $99 \div 9 = ?$ $\frac{1}{9}$ of $99 = ?$
4. $12 \times 9 = ?$ $108 \div 9 = ?$ $\frac{1}{9}$ of $108 = ?$

Continue the drill.

5. Subtract by 9's from 108.
6. Twelve 9's = ? In 81 how many 9's?
7. Nine 9's = ? In 108 how many 9's?
8. Eleven 9's = ? In 99 how many 9's?
9. Ten 9's = ? In 63 how many 9's?
10. $6 \times 9 = ?$ $9 \times 5 = ?$ $\frac{1}{9}$ of $81 = ?$
11. $63 \div 7 = ?$ $54 \div 9 = ?$ $\frac{1}{11}$ of $99 = ?$

Continue the drill.

69. ORAL

1. Nine times ten? Nine times eleven? Nine times twelve?

2. One ninth of 63; 72; 81; 108?

3. Two ninths of 18? Three ninths of 18?

4. 6 is what part of 18? 12 is what part of 18?

5. One third of 18 is how many ninths of it?

6. One ninth of 27? Three ninths of 27? One third of 27?

7. One third of 27 equals how many ninths of it?

8. How may you find three ninths of a number without dividing by 3? Without dividing by 9?

Three ninths of a number equal one third of it. To find three ninths of a number, divide the number by 9 to find one ninth of it; multiply the quotient by 3 to find three ninths of it. Or, divide the number by 3 to find one third of it, which is three ninths of it.

9. Two eighths of a number equal how many fourths of it? One half of a number equals how many fourths of it? How many eighths of it?

Learn:

One half of a number equals two fourths of the number.
One half of a number equals four eighths of the number.
One half of a number equals three sixths of the number.
One third of a number equals three ninths of the number.
One fourth of a number equals three twelfths of the number.

10. How many is one third of 108? How many are five ninths of 36?

11. If you have $\frac{2}{9}$ of a number given, how can you find one ninth of that number?

12. John has ten marbles. John's marbles are $\frac{2}{9}$ as many as those James has. How many marbles has James?

13. What sets of two numbers multiplied together give 18? 27? 36? 54? 63? 72? 81?

70. REVIEW

Divide:

1. $9\overline{)108}$ $9\overline{)106}$ $12\overline{)104}$ $12\overline{)78}$ $12\overline{)85}$ $8\overline{)73}$

2. The quotient is 5; the divisor is 7. What is the dividend?

3. The dividend is 84; the divisor is 7. What is the quotient?

4. If the dividend is divided by the quotient, what will the result equal? Make the complete statement. Give an example showing this.

What is a divisor? A dividend? A quotient? A sum? A product?

Learn: A number that will contain another number exactly is called a **multiple** of that number.

16 is a multiple of 8. 24 is a multiple of 8. Give other illustrations.

5. Give in order, as far as you can, the other multiples of 8.

6. What is the largest multiple of 9 you have learned? The smallest? Give other multiples between these two.

7. Give the multiples of 8 in order, beginning with 8.

8. 16 is a multiple of what other numbers besides 8?

9. 12 is a multiple of what numbers?

10. 20 is a multiple of what numbers?

11. 24 is a multiple of what numbers?

PROBLEMS

71.

WRITTEN

1. A carriage dealer sells 9 carriages at $ 87 apiece, and 7 wagons at $ 75 each. How much does he receive for the carriages and wagons?

Make another question. Answer it.

2. A real estate dealer sells five lots for $ 1045 and four lots for $ 1304. How much does he receive for the nine lots?

3. What is the average price he receives for each lot? Make a similar problem using small numbers.

4. If an ocean steamer sails an average of 425 miles a day, how far will it sail in five days? Solve in two ways.

5. If an errand boy earns $ 18 a month and spends $ 7 a month, how long will it take him to save $ 99?

Analysis: If he earns $18 a month and spends $7 a month, he saves $11 a month. It will take him as many months to save $99 as 11 is contained times in 99, or —— months.

6. A bicycle wheel is 9 feet in circumference. How far will the bicycle have gone when the wheel turns around once, or makes one revolution? How far will the rider go while the wheel is turning around 116 times?

7. How many revolutions will the wheel make in traveling 621 feet?

8. A boy buys a bushel of berries for one dollar, and sells them at 28 cents a peck. How much does he gain? Analyze by question and answer.

9. A merchant spends $21 for cotton goods and twice as much for woolen goods. Make two problems from these conditions. Solve each.

10. A grocer buys 98 barrels of apples at $2.55 a barrel and sells them at $3 a barrel. What is his gain? Work the shortest way.

11. If a dealer pays $348 a dozen for suits of ready-made clothing, how much does he pay for 9 suits? What is the first question to be answered?

To the Teacher. See note, p. 110.

12. Divide each of the following numbers by 9, giving the quotients and remainder in each case: 20, 24, 28, 30, 38, 40, 46, 64, 48, 73, 57, 65, 82, 88, 93, 100, 102, 107, 96, 86, 76.

FRACTIONAL PARTS OF NUMBERS

72. ORAL

1. John is 16 years old and Henry's age is $\frac{3}{4}$ of John's age. How old is Henry?

ANALYSIS: Henry's age is $\frac{3}{4}$ of 16 years. $\frac{1}{4}$ of 16 years is 4 years; $\frac{3}{4}$ of 16 years are 3 times 4 years, or 12 years.

This analysis may also be used: Henry's age is as many years as $\frac{3}{4}$ of 16. $\frac{1}{4}$ of $16 = 4$. $\frac{3}{4}$ of $16 = 3 \times 4 = 12$. Henry is 12 years old.

2. Henry's age is 12 years, which is $\frac{3}{4}$ of John's age. How old is John?

ANALYSIS: If 12 years are $\frac{3}{4}$ of John's age, $\frac{1}{4}$ of John's age is $\frac{1}{3}$ of 12 years, or 4 years. If 4 years is $\frac{1}{4}$ of John's age, $\frac{4}{4}$ of his age are 4 × 4 years, or 16 years.

TO THE TEACHER. Draw an oblong, 2 inches by 12 inches, and divide it into inch squares. Require pupils to show that $\frac{1}{2}$ of the whole number of squares equals $\frac{4}{8}$ of the whole number; $\frac{3}{6}$ of the whole number; $\frac{6}{12}$ of the whole number; that $\frac{1}{4}$ of the whole number of squares equals $\frac{2}{8}$ of the whole number; $\frac{3}{12}$ of the whole number.

3. There are ten lines in a row. The number in this row is $\frac{5}{6}$ of the number in the second row. How many lines are there in the second row? Make the two rows of lines and show that the two statements in the problem and the answer are correct.

4. Show that $\frac{2}{6}$ of 12 lines is the same as $\frac{1}{3}$ of 12 lines.

5. Show that $\frac{3}{6}$ of 12 lines is the same as $\frac{1}{2}$ of 12 lines.

6. Show that $\frac{4}{6}$ of 12 lines is the same as $\frac{2}{3}$ of 12 lines.

7. Show that $\frac{2}{6}$ of 12 is the same as $\frac{1}{3}$ of 12.

$$\begin{array}{r} 6\underline{)12} \\ 2 \\ \underline{2} \\ 4 \end{array}$$

$\frac{1}{6}$ of 12 is found by dividing 12 by 6. The quotient is 2. 2 multiplied by 2 gives $\frac{2}{6}$ of 12, or 4.

$\frac{1}{3}$ of 12 is found by dividing 12 by 3. The quotient is 4.

$$\begin{array}{r} 3\underline{)12} \\ 4 \end{array}$$

We find that 4 is $\frac{2}{6}$ of 12 and also $\frac{1}{3}$ of 12. $\frac{1}{3}$ of 12 = $\frac{2}{6}$ of 12.

8. Find $\frac{2}{6}$ of 18. How many thirds of 18 does this number equal?

9. Find $\frac{3}{9}$ of 18. How many thirds of 18 does this number equal?

10. Name three fractional parts of 18 that are equal.

11. Find the same parts of 36 and see whether they are equal.

Learn:

$\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ of a number are equal. $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ are equal.

If we multiply the 1 and the 3 in $\frac{1}{3}$ by 2 we get $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$.
If we multiply the 1 and the 3 in $\frac{1}{3}$ by 3 we get $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$.
If we multiply the 1 and 2 in $\frac{1}{2}$ by 2, we get $\frac{2}{4}$.
If we multiply the 1 and 2 in $\frac{1}{2}$ by 3, we get $\frac{3}{6}$.

12. Show that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equal.

13. Show that $\frac{1}{2}$ of a foot, $\frac{2}{4}$ of a foot, and $\frac{3}{6}$ of a foot give the same number of inches.

14. Multiply both numbers in $\frac{2}{3}$ by 2, and by 3; show that these parts of 18 are the same as $\frac{2}{3}$ of 18.

Learn:

When two numbers are written one above, the other below a line, as, $\frac{3}{4}$, they are called a **fraction**.

The two numbers used in a fraction are called **terms** of the fraction.

The number above the line is the **numerator**.

The number below the line is the **denominator**.

15. What are the terms of the fractions:

$$\frac{4}{5} \qquad \frac{5}{6} \qquad \frac{7}{8} \qquad \frac{9}{10}$$

16. What are the numerators of the four fractions?

17. What are the denominators of the four fractions?

$\frac{2}{3}$ of 12 lines means that the 12 lines are divided into 3 groups and that 2 of these groups are taken, thus:

$$\underset{\text{2 groups} = 8}{\underline{\overset{\text{1 GROUP}}{\overbrace{1\;1\;1\;1}^{4}}\qquad\overset{\text{1 GROUP}}{\overbrace{1\;1\;1\;1}^{4}}}}\qquad\overset{\text{1 GROUP}}{\overbrace{1\;1\;1\;1}^{4}}$$

$\frac{2}{3}$ of 12. We divide 12 by 3 to divide the number into 3 equal parts. $\frac{1}{3}$ of 12 is 4. If we take 2 parts, we take 2 times 4, or 8.

The **denominator**, 3, shows into how many parts the number is divided.

The **numerator**, 2, shows how many of the parts are taken.

18. $\frac{4}{5}$ of a number shows that the number is to be divided into —— parts, and that —— parts are to be taken. $\frac{4}{5}$ of 20 = ?

TABLE OF TENS

73. Add by 10's from 10 to 100.

Learn :

$10 \times 1 = 10$
$10 \times 2 = 20$
$10 \times 3 = 30$
$10 \times 4 = 40$
$10 \times 5 = 50$
$10 \times 6 = 60$

$10 \times 7 = 70$
$10 \times 8 = 80$
$10 \times 9 = 90$
$10 \times 10 = 100$
$10 \times 11 = 110$
$10 \times 12 = 120$

74. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $100 \div 10 = ?$ $\frac{1}{10}$ of $100 = ?$ $\frac{3}{10}$ of $100 = ?$
2. $110 \div 10 = ?$ $\frac{1}{10}$ of $110 = ?$ $\frac{1}{11}$ of $110 = ?$
3. $120 \div 10 = ?$ $\frac{1}{10}$ of $120 = ?$ $\frac{1}{12}$ of $120 = ?$
4. Subtract by 10's from 120.

75. ORAL

1. How many are 10×10? 12×10? 11×10? 10×12?

2. How many 10's are there in 80? in 90? 70? $\frac{3}{10}$ of 50? $\frac{5}{10}$ of 70? $\frac{8}{10}$ of 80?

3. $\frac{6}{10}$ of 30? $\frac{1}{2}$ of 20? $\frac{5}{10}$ of 20? $\frac{1}{2}$ of 20 = what other fractional part of 20?

$\frac{1}{2}$ of any number is $\frac{5}{10}$ of that number.

4. What is $\frac{1}{2}$ of 30? 40? 50? 60? 70? 80? 90? 100? 110? 120? 20 is what part of 40?

5. What is $\frac{1}{10}$ of 20? $\frac{2}{10}$ of 20? $\frac{1}{5}$ of 20?

6. $\frac{1}{5}$ of 20 = how many 10ths of it? $\frac{1}{5}$ of 30 = how many 10ths of it? 6 = how many 10ths of 30?

7. $\frac{1}{5}$ of 40 = what? How many 10ths of 40 = 8?

8. $\frac{2}{10}$ of a number = what part of it?

9. How do you find $\frac{1}{5}$ of a number? How do you find $\frac{2}{10}$ of a number?

10. How may you find $\frac{1}{5}$ of 90 without dividing it by 5?

Multiply each of the numbers below by 10, writing the products:

1 × 10 = 10
2 × 10 = 20
3
4
5
6
7
8
9
10

Compare the products with the numbers multiplied, and tell what has been done in each case.

11. To multiply a number by 10, what must you do? If you divide each of these products by 10, what will your result be? What have you done in each case?

NOTATION AND NUMERATION TO MILLIONS

76. **Study Recitation**

10 units = 1 ten, or 10
10 tens = 1 hundred, or 100
10 hundreds = 1 thousand, or 1,000
10 thousands = 1 ten thousand, or 10,000
10 ten thousands = 1 hundred thousand, or 100,000
10 hundred thousands = 1 million, or 1,000,000

Numbers in the first place are **units**; in the second place, **tens**; in the third, **hundreds**; in the fourth, **thousands**; in the fifth, **ten thousands**; in the sixth, **hundred thousands**; in the seventh, **millions**.

Numbers of more than three figures are pointed off as above; the first three figures make the first **period** (*units*), the next three, the second **period** (*thousands*), and so on.

To read numbers, begin at the left, read the number in that period as though it stood alone, and give it the name of the first place in the period, which is the name of the period. Read the next period at the right in the same way, and so on.

Thousands Units
1,111

The number is read, one *thousand* one hundred eleven. In reading numbers, the name of the first period at the right is usually omitted.

Thousands Units
$\underbrace{10},\underbrace{789}$

The number is read, ten *thousand* seven hundred eighty-nine.

Millions Thousands Units
$\underbrace{2},\underbrace{846},\underbrace{257}$

The number is read, two *million* eight hundred forty-six thousand, two hundred fifty-seven.

Read the following numbers, giving the number of *millions* in the third period, the number of *thousands* in the second period, and the number of *units* in the first period, but omit the name *units*.

Do not use *and* in reading these numbers.

Millions Hundred-Thousands Ten-Thousands Thousands Hundreds Tens Units	Millions Hundred-Thousands Ten-Thousands Thousands Hundreds Tens Units	Millions Hundred-Thousands Ten-Thousands Thousands Hundreds Tens Units
1,000	1,111	1,201
2,000	2,222	2,304
3,000	3,333	4,056
4,000	4,321	5,078
5,000	5,432	6,720
6,000	6,123	7,040
7,000	7,234	8,007
8,000	8,345	9,006
9,000	9,567	10,007
10,000	10,789	10,017
57,000	57,456	57,025
125,000	125,140	125,010
268,000	268,400	268,002
2,000,000	2,346,257	2,563,004
Millions Thousands Units	Millions Thousands Units	Millions Thousands Units

Write the following numbers:

To the Teacher. Have each number of more than three figures separated into periods by commas.

1. Nine hundred eight.
2. Seven hundred fifty-six.
3. Nine thousand, two hundred forty-three.
4. Eight thousand, five hundred sixty-four.
5. Twenty-seven thousand, three hundred eight.
6. Fifty-four thousand, five hundred six.
7. One hundred twenty-three thousand, two hundred seven.
8. Three hundred four thousand, two hundred.
9. Three hundred twenty-five thousand, six.
10. Seventy-eight thousand, seven.
11. One hundred ten thousand, three hundred eight.
12. Five hundred ten thousand, nine.

Write the following numbers in columns, placing the units of the same name in the same column and find their sum:

13. Four thousand, five hundred twenty-eight.
14. Five hundred ninety-six.
15. Three hundred seven.
16. Five thousand, eight hundred twenty-four.
17. Six thousand, three hundred nine.
18. Seventy-five.
19. Three hundred seventy-five.
20. Four thousand, three hundred seventy-five.

21. Eight.
22. Two hundred four.
23. Five thousand, nine.
24. Six hundred two.
25. One thousand, one.
26. Two thousand, six.

77.

Oral

1. If I buy 10 dozen eggs at 2¢ apiece, how much do I pay for them?

2. In $\frac{5}{10}$ of an hour how many minutes are there?

3. In $\frac{1}{2}$ hour how many minutes are there?

4. $\frac{1}{2}$ hour = how many 10ths of an hour?

5. $\frac{1}{2}$ hour = how many 12ths of an hour?

6. $\frac{1}{2}$ hour = how many 6ths of an hour?

7. $\frac{1}{2}$ hour = how many 4ths of an hour?

8. If 10 tops cost $1, how much will 5 tops cost? 4 tops? 8 tops?

9. A man travels 120 miles in 10 hours. How far can he travel in 7 hours?

10. If he travels 60 miles in 5 hours, how long will it take him to travel 84 miles?

11. 30 is $\frac{3}{10}$ of what number? 7 is $\frac{7}{10}$ of what number?

12. 7 dimes are how much more than 50 cents? 7 dimes are how much less than $\frac{3}{4}$ of a dollar? How much less than a dollar? How much more than a quarter of a dollar?

13. Write the following numbers in proper order for addition, and add:

Forty-eight thousand, two hundred fifty-six.
Five hundred sixteen thousand, three hundred forty.
Two hundred seventy-five thousand, eight hundred fifty.
Twenty-seven thousand, nineteen.

78. WRITTEN

1. Divide:

$10\underline{)270}$ $10\underline{)340}$ $10\underline{)1620}$ $10\underline{)1720}$ $10\underline{)620}$

2. $10\underline{)840}$ $10\underline{)920}$ $10\underline{)1530}$ $10\underline{)2460}$ $10\underline{)580}$

Cut off the units' figure and write the remaining figures for the quotient.

3. Divide:

$10\underline{)275}$

Cut off the units' figure, 5. Write it over the divisor, 10, at the right of the remaining figures for the quotient. $27\frac{5}{10}$.

In the same way divide the following numbers by 10:

4.	**5.**	**6.**	**7.**	**8.**	**9.**
348	567	285	342	1684	278
10.	**11.**	**12.**	**13.**	**14.**	**15.**
564	972	813	568	1745	1839

TABLE OF ELEVENS

79.

$11 \times 1 = 11$
$11 \times 2 = 22$
$11 \times 3 = 33$
$11 \times 4 = 44$
$11 \times 5 = 55$
$11 \times 6 = 66$
$11 \times 7 = 77$
$11 \times 8 = 88$
$11 \times 9 = 99$
$11 \times 10 = 110$
$11 \times 11 = 121$
$11 \times 12 = 132$

80. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1. $121 \div 11 = ?$ $\frac{1}{11}$ of $121 = ?$ $\frac{5}{11}$ of $121 = ?$
2. $132 \div 11 = ?$ $\frac{1}{11}$ of $132 = ?$ $\frac{1}{12}$ of $132 = ?$
3. Subtract by 11's from 132.
4. Twelve 11's = ? In 132 how many 11's?
5. Eleven 11's = ? In 121 how many 11's?
6. $132 \div 12 = ?$ $99 \div 9 = ?$ $\frac{1}{11}$ of $121 = ?$
7. $110 \div 10 = ?$ $88 \div 11 = ?$ $\frac{1}{11}$ of $88 = ?$
8. $11 \times 10 = ?$ $99 \div 11 = ?$ $\frac{1}{11}$ of $77 = ?$

81. WRITTEN AND ORAL

In multiplying by 11 do not multiply by 1 unit and then by 1 ten, but multiply by 11 as though it were a single figure:

1.	2.	3.	4.	5.	6.
256	384	472	9286	4752	3402
11	11	11	11	11	11

7. Divide the following numbers by 11: 792, 869.

8. Multiply the following numbers by 12, using 12 as though it were a single figure: 274, 356, 599, 684.

9. Divide the following numbers by 12: 864, 372.

10. Find $\frac{3}{12}$ of 864; of 372; of 996.

11. If 11 horses cost $1386 and 12 cows cost $408, how much more do 2 horses cost than 4 cows? What is the cost of one horse? Of one cow?

12. The difference between $\frac{1}{5}$ of a number and $\frac{4}{5}$ of the number is how many 5ths of that number?

13. The difference between $\frac{8}{11}$ of a number and $\frac{5}{11}$ of the number is how many 11ths of that number?

14. The sum of $\frac{3}{10}$ of a number and $\frac{4}{10}$ of the number is how many 10ths of that number?

15. Find the sum of $\frac{3}{10}$ of 80 and $\frac{4}{10}$ of 80 in two ways.

16. $\frac{5}{11}$ of 132 are how many more than $\frac{3}{11}$ of the same number? Work in two ways.

17. 21 is $\frac{3}{11}$ of what number?

18. A farmer has 2 fields; one containing 20 acres is $\frac{5}{8}$ as large as the other. What is the size of the other field?

ANALYSIS: If a field of 20 acres is $\frac{5}{8}$ of another field, $\frac{1}{8}$ of the other field is $\frac{1}{5}$ of 20 acres, or 4 acres. $\frac{8}{8}$ or the whole of the other field is 8 times 4 acres, or —— acres.

19. A farmer has 680 bushels of oats in a bin and the number of bushels of oats is $\frac{5}{12}$ of the number of bushels of wheat in another bin. How many bushels of wheat has he?

82. TABLE OF TWELVES

12 × 1 = 12
12 × 2 = 24
12 × 3 = 36
12 × 4 = 48
12 × 5 = 60
12 × 6 = 72
12 × 7 = 84
12 × 8 = 96
12 × 9 = 108
12 × 10 = 120
12 × 11 = 132
12 × 12 = 144

83. TABLE FOR DRILL IN GIVING PRODUCTS AND QUOTIENTS RAPIDLY

1.	12 × 12 = ?	144 ÷ 12 = ?	$\frac{1}{12}$ of 144 = ?
2.	Eight 12's = ?	96 ÷ 12 = ?	$\frac{1}{12}$ of 84 = ?
3.	Six 12's = ?	132 ÷ 12 = ?	$\frac{1}{9}$ of 108 = ?
4.	Eleven 12's = ?	72 ÷ 6 = ?	$\frac{1}{12}$ of 60 = ?
5.	Four 12's = ?	36 ÷ 12 = ?	$\frac{1}{2}$ of 24 = ?
6.	Subtract by 12's from 144.		
7.	Subtract by 12's from 137.		
8.	Five 12's = ?	24 ÷ 12 = ?	$\frac{1}{3}$ of 36 = ?
9.	Seven 12's = ?	108 ÷ 12 = ?	$\frac{1}{10}$ of 120 = ?
10.	Two 12's = ?	84 ÷ 12 = ?	$\frac{1}{8}$ of 96 = ?
11.	Nine 12's = ?	60 ÷ 12 = ?	$\frac{1}{12}$ of 72 = ?
12.	Three 12's = ?	48 ÷ 12 = ?	$\frac{1}{12}$ of 96 = ?
13.	Ten 12's = ?	120 ÷ 12 = ?	$\frac{1}{12}$ of 132 = ?

FRACTIONAL PARTS OF NUMBERS

84. ORAL

1. What is a multiple of a number? Give the multiples of 12 as learned in the multiplication table.

2. Give $\frac{1}{2}$ of 24, 60, 48, 72, 144, 84, 96, 132, 72, 36. Give the quotients without writing the numbers. Think 2 in 2, 1; 2 in 4, 2; say 12.

3. What is $\frac{1}{12}$ of 24? $\frac{2}{12}$ of 24? 4 is how many sixths of 24? $\frac{1}{6}$ of 24 = how many 12ths of 24?

4. $\frac{1}{6}$ of 36 = ? $\frac{1}{12}$ of 36 = ?

5. $\frac{1}{6}$ of a number = how many 12ths of it?

6. How do you find $\frac{1}{12}$ of a number?

7. How may you find $\frac{1}{6}$ of a number without dividing by 6?

8. How much is $\frac{1}{6}$ of 144? How much is $\frac{1}{6}$ of 132?

9. What are $\frac{3}{12}$ of 24? This number = how many fourths of 24? Prove that $\frac{1}{4}$ of 48 = $\frac{3}{12}$ of 48.

10. Take 2 groups of 12 objects each and show that $\frac{3}{12}$ of one group = $\frac{1}{4}$ of the other group. Multiply both terms of the fraction $\frac{1}{4}$ by 3. What is the result?

11. What are $\frac{6}{12}$ of 24? What is $\frac{1}{2}$ of 24? $\frac{6}{12}$ of 24 = how many halves of 24? How can you change $\frac{1}{2}$ to $\frac{6}{12}$?

12. Show that $\frac{6}{12}$ of any number = $\frac{1}{2}$ of it.

13. $\frac{8}{12}$ of a number are how many times $\frac{4}{12}$ of it? $\frac{4}{12}$ of a number are how many thirds of it? $\frac{8}{12}$ of a number are how many thirds of it?

REVIEW

85.

WRITTEN

1. Multiply the following by 12, using 12 as though it were a single digit number: 2456, 8723, 10254, 5678, 7294.

2. Divide each of the following numbers by 12: 48, 49, 50, 54, 58, 60, 72, 64, 68, 70, 84, 129, 134, 144.

3. In 276 months how many years are there?

4. A box contains 288 oranges. If they are sold for 25¢ a dozen, how much will be received for them?

5. A man receives a salary of $1000 a year. He pays $18 a month for a house, $35 a month for meat and groceries, $200 a year for clothing, $100 for other expenses. What are his entire expenses? How much has he left at the end of the year?

6. A man sells 12 horses for $1020, and 9 cows for $315. How much more does he receive for a horse than for a cow?

7. I buy at a grocery 12 pounds of sugar at 6¢ a pound, 4 dozen eggs at 15¢ a dozen, and 6 oranges at 40¢ a dozen. What is the amount of my bill?

8. I bought a horse for $125. I sold it at a loss of $15. How much did I get for it?

9. How many dozen eggs, worth 12¢ a dozen, will pay for 20 pounds of sugar worth 7¢ a pound, and a gallon of sirup worth 76¢?

10.

6	3
4	8
9	10
12	5
2	11
7	1

To the Teacher. Drill pupils in multiplying each of the numbers in the two columns at the left by numbers from 1–12 inclusive. Write the numbers on the board and write the multiplier at the left of each column, then conduct the drill by pointing to different numbers in each column, requiring pupils to give instantly the product of the number pointed out by the number at the left. By changing the numbers at the left, all possible combinations of the multiplication tables may be given.

Give products rapidly:

11.	6	9	10	11	12	7	12	9	11	8	12
	8	7	5	4	5	3	7	12	8	9	6
12.	7	10	9	8	9	10	9	11	9	7	6
	11	8	6	5	5	9	4	9	3	8	5

Name:

13. The multiples of 2 to 100.
14. The multiples of 3 to 100.
15. The multiples of 4 to 100.
16. The multiples of 5 to 100.
17. The multiples of 6 to 100.
18. The multiples of 7 to 100.
19. The multiples of 8 to 100.
20. The multiples of 9 to 108.
21. The multiples of 10 to 120.
22. The multiples of 11 to 132.
23. The multiples of 12 to 144.

To the Teacher. In conducting these exercises, put any set of multiples on the board in irregular order, placing the divisor at the left, and have pupils give the quotients from the top down and from the bottom up.

Teacher or some pupil point to different multiples and call upon individuals to give the results. Drill for speed and accuracy.

Give drill in dividing numbers to 100 by numbers to 12 where the divisor is not exact. Require the work to be mental and oral.

SHORT DIVISION

86. In division, when the divisor is less than 13, the work is done mentally and not written out in full.

The following are examples in Short Division.

Test your work by multiplication:

1. Divide 25650 by 5.
2. Divide 24348 by 8.
3. Divide 46280 by 9.
4. Divide 36474 by 6.
5. Divide 31260 by 6.
6. Divide 36470 by 7.
7. Divide 20544 by 8.
8. Divide 11106 by 9.
9. Divide 50325 by 5.
10. Divide 35552 by 4.
11. Divide 29997 by 3.
12. Divide 43632 by 8.
13. Divide 37251 by 9.
14. Divide 23643 by 9.
15. Divide 45424 by 8.
16. Divide 78440 by 4.
17. Divide 39426 by 3.
18. Divide 27426 by 7.
19. Divide 42376 by 2.
20. Divide 56840 by 10.
21. Divide 38604 by 12.
22. Divide 46387 by 11.
23. Divide 33330 by 11.
24. Divide 45912 by 12.
25. Divide 62070 by 10.
26. Divide 13574 by 11.
27. Divide 40200 by 10.
28. Divide 37476 by 12.
29. Divide 45122 by 11
30. Divide 49224 by 12.

To THE TEACHER. For further drill, require pupils to divide each or all of the above numbers by each of the numbers from 2 to 12 inclusive. To secure rapidity in dividing, it is well to have pupils give the partial quotients orally. Indicate the division on the board, thus: $5\overline{)4685}$. Pupils give orally, nine, three, seven; quotient, nine hundred thirty-seven.

LONG DIVISION

87. Long Division is used when the work is all written.

Study Recitation

1. Divide 16821 by 9

(*a*) Short Division form:

$$\begin{array}{r} 9\underline{)16821} \\ 1869 \end{array}$$

(*b*) Long Division form:

$$\begin{array}{r} 1869 \\ 9\overline{)16821} \\ \underline{9} \\ 78 \\ \underline{72} \\ 62 \\ \underline{54} \\ 81 \\ \underline{81} \end{array}$$

In the Long Division form, we write the divisor and dividend as in Short Division. We find the quotient figure 1, and write it above the 6, instead of below, as in short division. We then multiply the divisor 9 by 1 and write the product 9 under 16, the part of the dividend used, and subtract it from 16, writing the remainder, 7, below. To the remainder 7 we annex 8, the next figure in the dividend, making 78. We say, 9 in 78, 8 times, and write the 8 over the 8 in the dividend. We then multiply the divisor 9 by the 8, and write the product 72 below 78, and subtract. We annex to the remainder, 6, the next figure of the dividend 2, making 62. We say, 9 in 62, 6 times, and write the 6 over the 2 in the dividend. We then multiply the divisor 9 by the 6, and subtract the product 54 from 62. To the remainder 8, we annex 1, the last figure in the dividend, making 81. We say, 9 in 81, 9 times, and write the 9 over the 1 in the dividend. We then multiply the divisor 9 by the last figure in the quotient, or 9, and write the product 81 under the 81. As there is no remainder, the division is exact.

To THE TEACHER. Pupils should see clearly that the mental work is exactly the same in both forms of division; but that in the Long Division form, the work of multiplying, subtracting, and annexing is all written out, while in the Short Division form the results are carried in the mind without writing anything except the quotient figures.

In Long Division it is best to write each of the quotient figures *over* the last figure of the partial dividend used to produce it, as there is less liability of error in bringing down the proper figure for the next partial dividend. This is where beginners often make mistakes.

Solve the following by both Short Division and Long Division.

Learn: The remainder after each partial division must be less than the divisor.

Divide:

2. 8)2760

3. 7)46844

4. 9)4275

5. 6)3624

6. 11)27984

7. 12)98904

8. 12)6576

9. 11)2365

After the Long Division form is learned, it should never be used when the divisor is 12 or less.

88. To find the quotient figures in Long Division when the second figure from the left in the divisor is small:

Study Recitation

Divide:

$$\begin{array}{r} 2 \\ 1.\quad 21\overline{)42} \\ 42 \\ \hline \end{array}$$

Divide the left-hand figure in the dividend by the left-hand figure in the divisor. 2 in 4, 2 times. Write 2 for the quotient figure and multiply. There is no remainder.

2. $$\begin{array}{r} 2\\ 31\overline{)74}\\ 62\\ \hline 12 \end{array}$$

In the same way: 3 in 7, 2 times. Write 2 for the quotient figure and multiply. The remainder 12 is less than the divisor.

3. $$\begin{array}{r} 2\\ 61\overline{)157}\\ 122\\ \hline \end{array}$$

As the first figure at the left of the dividend will not contain the divisor, divide 15 by 6. 6 in 15, 2 times. The last figure in the partial dividend is 7. Write 2 for the quotient figure over the 7 and multiply. The remainder is less than the divisor.

Practice

Divide each of the numbers in column *A* by the number at its left, each number in column *B* by the number at its left, and so on, to find the quotient figure in each case. Subtract the product of the quotient figure by the divisor, from the dividend, to see if the remainder in each case is less than the divisor.

	A.		*B.*		*C.*		*D.*
	4.		**5.**		**6.**		**7.**
a.	21)42	*a.*	31)74	*a.*	41)87	*a.*	51)127
b.	65	*b.*	65	*b.*	69	*b.*	235
c.	87	*c.*	86	*c.*	54	*c.*	478
d.	93	*d.*	93	*d.*	49	*d.*	374
e.	75	*e.*	72	*e.*	78	*e.*	185
f.	64	*f.*	46	*f.*	96	*f.*	278
g.	53	*g.*	59	*g.*	82	*g.*	196

89. To find the first partial dividend, the first quotient figure, and its place.

Study Recitation

Learn: The first partial dividend has as many figures in it as there are figures in the divisor, or one more.

If the divisor is greater than the number represented by the same number of figures on the left of the dividend, the first partial dividend contains one more figure than the divisor; otherwise, the same number.

Before beginning the division in any case, see what figures make up the first partial dividend.

Divide:

1. $$\begin{array}{r} 7 \\ 21\overline{)1598} \end{array}$$

The divisor 21 is greater than 15, the number represented by the first two figures on the left of the dividend. There are two figures in the divisor; there are three figures in the first partial dividend, or 159. Place the first figure of the quotient over 9, the last figure in the partial dividend.

2. $$\begin{array}{r} 5 \\ 567\overline{)29817} \end{array}$$

567 is greater than 298. There are three figures in the divisor and four in the first partial dividend, or 2981. Place the first figure of the quotient over 1, the last figure of the partial dividend.

3. $$\begin{array}{r} 2 \\ 356\overline{)7832} \end{array}$$

356 is not greater than 783. There are three figures in the divisor, and three in the first partial dividend. Place the first figure of the quotient over 3, the last figure in the partial dividend.

4. 423)17766 How many figures in the partial dividend?

5. 326)2282 How many figures in the partial dividend?

6. 407)15466 How many figures in the partial dividend?

7. $62\overline{)9114}$ Where should the first figure in the quotient be placed?

8. $54\overline{)1242}$ Where should the first figure in the quotient be placed?

9.

$$\begin{array}{r} 55 \\ 52\overline{)2860} \\ \underline{260} \\ 260 \\ \underline{260} \end{array}$$

What is the first partial dividend?

5 is contained in 28, 5 times. Write 5 for the first quotient figure over 6, the last figure in the partial dividend. Multiply 52 by 5, and write the product under the partial dividend. Subtract. To the remainder, 26, annex 0, the next figure in the dividend. 5 is contained in 26, 5 times. Write 5 for the second figure in the quotient. Multiply 52 by 5 and subtract. The remainder is 0. 55 is the quotient.

Practice

10. $2646 \div 42 = ?$

11. $3402 \div 63 = ?$

12. $1924 \div 74 = ?$

13. $3735 \div 83 = ?$

14. $1504 \div 94 = ?$

15. $4745 \div 73 = ?$

16. $1820 \div 52 = ?$

17. $3402 \div 54 = ?$

18. $2646 \div 63 = ?$

19. $1472 \div 64 = ?$

20. $2928 \div 61 = ?$

21. $3534 \div 62 = ?$

22. $2277 \div 33 = ?$

23. $2438 \div 53 = ?$

24. $1958 \div 24 = ?$

25. $3116 \div 41 = ?$

26. $1176 \div 42 = ?$

27. $3726 \div 54 = ?$

28. $2856 \div 34 = ?$

29. $5670 \div 70 = ?$

30. $1725 \div 23 = ?$

31. $5096 \div 52 = ?$

90. To find the quotient figures when the second figure from the left in the divisor is large—7, 8, or 9:

Drill preparatory to work in finding Quotient Figures in Long Division.

Divide the numbers in each column by the number at its left, naming the quotient and remainder in each case:

Multiply the numbers in each column by the number at its left, giving the products orally without written work:

4)14	5)17	2)19	2 × 24	3 × 18	4 × 16	5 × 17
19	27	17	19	27	27	19
17	36	15	27	46	35	29
27	49	11	36	35	46	39
35	24	18	15	57	52	34
29	28	23	17	29	64	52
33	46	22	13	19	75	66
23	43	17	25	46	82	73
15	37	16	39	37	93	85

Study Recitation

When the second figure in the divisor is 9, increase the first figure by 1 before dividing. The number obtained by dividing by the figure increased by 1 will nearly always be the correct quotient figure. In a few cases it will be 1 too small, and the next larger number must be taken for the quotient figure.

Study Recitation

Divide:

1. $34\frac{17}{19}$
19)663
57
93
76
17

As the second figure in the divisor is 9, increase the first figure by 1, making it 2. Divide the first figure in the dividend by 2. The quotient is 3. Place 3 above the 6 for the first quotient figure. Multiply, subtract, bring down the next figure, and divide the 9 by 2. The result is 4. Multiply the divisor by 4, and subtract. The remainder is found to be 17, which is written over the divisor at the right of 4, and read seventeen nineteenths.

2. 42
18)756
72
36
36

As the second figure in the divisor is 8, increase the first figure by 1. Divide the first figure in the dividend by 2. The quotient is 3.

Multiply 18 by 3, and subtract the product, 54, from 75 (this should be done mentally and without writing the product or remainder). As the remainder, 21, is greater than the divisor, the quotient figure is too small. Increase it by 1, making 4. Complete the division.

Practice

Divide the numbers in each column by the number at its left. Write the remainder over each divisor at the right of the quotient figure:

19)	35	19)	41	19)	39	29)	65	49)	96
	47		55		59		83		89
	65		62		38		95		256
	72		71		77		156		365
	84		88		97		214		278
	94		33		78		185		197

3. Divide 65, 83, 95, 156, by 29; by 28.

Use 30 for a trial divisor instead of 29 and 28.

4. Divide 86, 95, 127, 139, by 39; by 38.
5. Divide 96, 89, 156, 365, by 49; by 48.
6. Divide 158, 147, 162, 194, by 19; by 18.
7. Divide 255, 165, 237, 249, by 79; by 78.
8. Divide 274, 326, 178, 258, by 69; by 68.
9. Divide 359, 178, 267, 554, by 89; by 88.
10. Divide 59, 63, 79, 84, by 29; by 28; by 27.
11. Divide 76, 85, 142, 157, by 39; by 38; by 37.
12. Divide 85, 135, 212, 99, by 49; by 48; by 47.
13. Divide 315, 175, 196, 360, by 28; by 27.

91. Examples in which zeros occur in the quotient:

Study Recitation

Divide:

1.
$$\begin{array}{r} 107 \\ 24\overline{)2568} \\ \underline{24} \\ 168 \\ \underline{168} \end{array}$$

The first quotient figure is 1. Multiplying, subtracting, and bringing down 6, we have 16 for the second partial dividend. As 24 is not contained in 16, place a zero over 6, the figure brought down, and bring down the 8, making the third partial dividend, 168. 7 is the third quotient figure, and there is no remainder.

Practice

2. $5100 \div 25 = ?$
3. $28908 \div 36 = ?$
4. $40948 \div 58 = ?$
5. $16224 \div 32 = ?$
6. $23115 \div 23 = ?$
7. $11457 \div 57 = ?$
8. $36252 \div 36 = ?$
9. $7105 \div 35 = ?$
10. $24928 \div 82 = ?$

92. Examples in which the last figure in the quotient is zero.

Study Recitation

Divide:

1.
$$\begin{array}{r} 260\frac{6}{48} \\ 48\overline{)12486} \\ \underline{96} \\ 288 \\ \underline{288} \\ 0006 \end{array}$$

After multiplying the divisor by the second figure, 6, of the quotient, there is no remainder, but there is another figure in the dividend. When the 6 is brought down, it is seen that it will not contain 48. Hence a zero is placed in the quotient over the 6; the 6 is written to the right of the quotient, with the divisor underneath.

Practice

2. $10754 \div 43$
3. $14260 \div 62$
4. $295687 \div 28$
5. $807240 \div 62$
6. $296675 \div 29$
7. $115507 \div 35$

93. To divide by 10, 100, or 1000.

Study Recitation

Divide:

1.
$$\begin{array}{r} 267\frac{8}{10} \\ 10\overline{)2678} \\ \underline{20} \\ 67 \\ \underline{60} \\ 78 \\ \underline{70} \\ 8 \end{array}$$

2.
$$\begin{array}{r} 26\frac{78}{100} \\ 100\overline{)2678} \\ \underline{200} \\ 678 \\ \underline{600} \\ 78 \end{array}$$

3.
$$\begin{array}{r} 2\frac{678}{1000} \\ 1000\overline{)2678} \\ \underline{2000} \\ 678 \end{array}$$

In example 1, what figures in the dividend are found in the quotient? Which figure of the dividend is the remainder?

In dividing a number by 10, the last figure is the remainder and the other figures are the quotient. In dividing by 100, what figures are the remainder and what figures the quotient? In dividing by 1000?

Practice

Divide each of these numbers by 10, 100, and 1000:

4. 2784
5. 5692
6. 2730
7. 56,957
8. 39,560
9. 4682
10. 57,940
11. 36,972

94. To divide when the divisor contains significant figures with 0 on the right.

Divide:

$$\begin{array}{rr}
 & 234 \\
1. & 20\overline{)4695} \\
 & \underline{40} \\
 & 69 \\
 & \underline{60} \\
 & 95 \\
 & \underline{80} \\
 & 15
\end{array}$$

$$\begin{array}{rr}
 & 234\tfrac{15}{20} \\
2. & 2|0\overline{)469|5}
\end{array}$$

Study Recitation

In example 1 the quotient is 234 and the remainder is 15. In example 2, before dividing, the zero at the right of the divisor is cut off, and the right-hand figure of the dividend is cut off. The remaining portion of the dividend is divided by 2, or what is left of the divisor after cutting off the zero. The remainder from the division by 2 is 1. Annex the figure cut off from the dividend to the 1, making the complete remainder 15.

If there are two 0's on the right of the divisor, cut them both off and cut off two figures from the right of the dividend before dividing.

Practice

3. $2356 \div 20$
4. $1498 \div 20$
5. $3572 \div 20$
6. $1843 \div 30$
7. $4652 \div 40$
8. $7875 \div 50$
9. $2563 \div 120$
10. $3764 \div 130$
11. $4563 \div 140$

95. To divide when the divisor contains more than two significant figures.

Study Recitation

Divide:

1.
```
       2457 27/53
   53)130248
      106
       242
       212
        304
        265
         398
         371
          27
```

2.
```
          243
   536)130248
       1072
        2304
        2144
         1608
         1608
```

In example 1, to find the first quotient figure divide the 13 on the left of the dividend by 5. The first quotient figure is 2. 53 is more than 13, so there are 3 figures in the first partial dividend, or 130. Write the quotient figure 2 over the last figure, or 0, in the partial dividend. Multiplying, subtracting, and bringing down the next figure in the dividend, we have 242 for the second partial dividend. The work is completed as in previous examples when the divisor is made up of two significant figures.

In example 2, the dividend is the same as in example 1. The divisor has three figures, the first two of which are the same as in example 1. To find the first quotient figure, proceed as in 1, dividing 13 by 5. The quotient figure is 2. 536 is more than 130, so there are four figures in the first partial dividend, or 1302. Write the quotient figure 2 over the last figure in the partial dividend. Multiplying, subtracting, and bringing down the next figure of the dividend, the second partial dividend is 2304. Divide the 23 by 5 for the second figure in the quotient, or 4. Multiplying, subtracting, and bringing down the next figure of the dividend, we find that the last partial dividend is 1608. Divide the 16 by 5 for the next figure of the quotient. It will be seen that the third figure, 6, in the divisor is not used in finding the trial quotient figures. If there were four or five figures in the divisor, the quotient figures would be found in most cases by using only the first two on the left.

Practice

3. 407,391 ÷ 687
4. 836,847 ÷ 963
5. 566,955 ÷ 645
6. 8300 ÷ 426
7. 6325 ÷ 523
8. 16,748 ÷ 4187

REVIEW

96. 1. 4536 ÷ 21
2. 11,904 ÷ 31
3. 32,004 ÷ 42
4. 41,676 ÷ 92
5. 28,607 ÷ 23
6. 3469 ÷ 62
7. 3481 ÷ 83
8. 20,708 ÷ 83
9. 27,068 ÷ 92
10. 5766 ÷ 74
11. 727,748 ÷ 98
12. 211,044 ÷ 86
13. 41,220 ÷ 45
14. 460,346 ÷ 58
15. 52,542 ÷ 63
16. 31,080 ÷ 35
17. 50,854 ÷ 94
18. 56,394 ÷ 78
19. 19,198 ÷ 37
20. 13,570 ÷ 54
21. 35,794 ÷ 43
22. 35,911 ÷ 28
23. 13,566 ÷ 57
24. 19,228 ÷ 38
25. 6939 ÷ 257
26. 14,504 ÷ 98
27. 19,836 ÷ 76
28. 62,505 ÷ 135
29. 104,328 ÷ 504
30. 266,688 ÷ 576

ORAL REVIEW AND GRAPHIC WORK

97. 1. A man earns $2 a day for six days in the week. He pays $4 a week for board and $3 a week for other expenses. How many dollars can he save in a week? In how many weeks can he save $30?

2. $8 + 6, \div 2, \times 8, - 6, \div 5, + 8, \div 6, + 21, \div 12 = ?$

When numbers are written, as in example 2, with a comma after each, perform the operations in order as indicated by the signs.

3. A man has a farm containing 80 acres. $\frac{1}{4}$ of the farm is a wood lot, $\frac{3}{8}$ of the farm is sown in wheat, $\frac{1}{8}$ of the farm is planted with corn, and the rest is planted with potatoes. Make and answer as many questions on the above as possible.

4. How many inches in 8 feet (ft.) and 9 inches (in.)?

5. A room is 6 yards (yd.) 2 ft. long, and 15 ft. wide. The floor is carpeted with strips of carpet 1 yd. wide, running lengthwise of the floor. How many feet long is each strip? How many strips will it take to cover the floor? How many yards of carpet does it take to cover the floor? Draw the floor on a scale of 1 inch to the foot, and show by dotted lines where the strips of carpet are joined.

6. A garden is 3 rods wide and 5 rods long. Draw a figure on the board to represent the garden. Divide the figure so as to represent lots 1 rod square. How many such lots are there in the garden? In $\frac{2}{3}$ of

the garden? In $\frac{1}{5}$ of the garden? In $\frac{3}{5}$ of the garden? What is the shortest way of finding the number of lots 1 yd. square in the garden?

7. Draw a rectangle 15 inches wide and 20 inches long. Draw lines to divide it into rectangles 3 inches wide by 4 inches long. How many such rectangles are there on one side? On one end? How many are there in all? How can you find the entire number without counting? How many 1-inch squares are there in each small rectangle? How many are there in the large rectangle?

8. A large rectangle is divided into 10 (50) equal small rectangles. There are 2 (5) of the small rectangles on one end. How many are there on each side? Draw the rectangle showing the 10 small rectangles.

9. The cover of an arithmetic is 7 in. long and 5 in. wide. How many square inches are there on one side? Show by a figure on the board.

10. If there are 35 square inches on one side of a book cover, and the cover is 5 in. wide, how long is it? How many square inches are there on both sides of the cover?

11. When you know the number of square inches in a rectangle, and its length in inches, how can you find its width in inches?

Oral and Written

98. 1. A dealer buys 8 cords of wood at $4 a cord, and sells it so as to gain $10. For how much does he sell it? What is the selling price per cord?

2. A dealer buys 65 horses at $75 each. He sells them so as to gain $325 on the lot. How much does he receive for the lot? How much does he get for each? Compare with problem 1.

3. A dealer sells 8 cords of wood for $42 and gains $10. How much does he pay for the wood? How much per cord? Compare with problem 1, and tell how it differs from that problem.

4. When the cost and selling price are known, how can you find the gain or loss? When the cost and gain are known, how can you find the selling price?

5. A dealer sells 65 horses for $78 each, and gains $5 on each horse. How much does he pay for the lot? When the selling price and gain are known, how can you find the cost price?

6. Find the cost of sending a telegram of 25 words from Chicago to New York, at 40¢ for the first ten words and 3¢ for each additional word.

7. How much will it cost to talk over a long-distance telephone for 9 minutes at 50¢ for the first 3 minutes and 15¢ for each additional minute?

8. A grocer buys 4 bu. of plums at $2.50 a bushel, and sells them at 10¢ a quart. How much does he gain? Solve by question and answer.

9. A grocer buys vinegar at 24¢ a gallon and sells it at 5¢ a pint. How much does he gain on one gallon? How much does he gain on 1 quart? How much does he gain on 10 gallons?

10. A grocer pays a jobber $15 a barrel for molasses. He pays $1 per barrel for freight and cartage. Each barrel contains 50 gallons. He sells it for 40¢ a gallon. How much does he make on a barrel? Solve by question and answer.

11. A field is 10 rods wide and 30 rods long. How many rods of wire will it take to put a 3-wire fence around the field?

12. A man earns $85 a month. His expenses are $720 a year. How much can he save in 2 years? This problem may be solved in several ways. See how many different solutions can be given by the class.

13. A mill makes 9212 pounds of flour a day. How many barrels of 196 pounds will it make in 6 days? How many barrels does it make in 1 day?

14. A's farm is worth $3200. B's farm is worth $\frac{5}{8}$ as much. How much are both farms worth? Analyze.

15. $\frac{2}{3}$ of $36 + \frac{3}{4}$ of 28 are how many?

16. $\frac{2}{3}$ of $351 + \frac{3}{4}$ of 728 are how many?

17. $\frac{3}{4}$ of 180 are how many more than $\frac{2}{3}$ of the same number?

18. A real estate dealer bought 3 houses, paying for them $3500, $2750, and $1975. He sold the three for $1825 more than they cost? What was his gain? For how much did he sell them? State the first question to be answered in solving the problem.

19. An ocean steamer goes 400 miles in one day; 460 miles a day for the next two days, and 428 miles a day for the next three days. What is the entire distance it goes in the six days?

20. A has $250; B has 2 times as much as A; C has $\frac{1}{3}$ as much as A and B. How much have they all?

21. $8 \times 6,\ +2,\ \div 5,\ +6,\ \div 4,\ \times 12,\ -12,\ \div 6,\ +9 = ?$

22. $24 + 9,\ +8,\ +4,\ \div 5,\ +8,\ -5,\ \times 12,\ -6,\ -8,\ -4 = ?$

23. $6 \times 7,\ +6,\ \div 8,\ \times 5,\ +3,\ \div 11,\ \times 20,\ \div 6,\ \times 10,\ -9 = ?$

Find sums:

24.	684	**25.**	$24.50	**26.**	$3.56	**27.**	$46.72
	396		37.64		7.28		52.12
	578		18.47		1.14		63.75
	493		9.72		9.63		48.09
	679		15.06		2.75		72.00
	584		27.33		.84		15.96

28. A farmer buys 24 cows for $864. What is the average price? Prove your work. Analyze.

29. John travels 48 miles in one day, and James travels $\frac{7}{8}$ as far. How far does James travel?

30. Change the last problem by putting a 2 before the 48, making it 248.

31. Change further by making the fraction $\frac{15}{31}$.

32. John has 24 marbles which is $\frac{3}{4}$ of the number James has. How many has James? Analyze.

33. There are 16 boys in a class. The number of boys is $\frac{4}{5}$ the number of girls in the class. How many girls are there in the class?

34. There are 16 boys in a class. The number of girls is $\frac{3}{4}$ the number of boys. How many girls are there in the class?

How does this problem differ from the one before it?

35. The width of a room is $\frac{3}{4}$ of its length. It is 24 feet long. How wide is it?

36. The width of a room is $\frac{3}{4}$ of its length. It is 18 feet wide. How long is it?

37. A room is 18 feet wide. The width is $\frac{3}{4}$ of the length. How long is it?

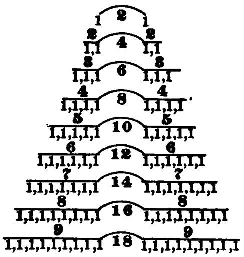
38. A train travels 40 miles an hour. How much farther does it travel in $\frac{4}{5}$ of an hour than in $\frac{3}{8}$ of an hour? Analyze.

FRACTIONS

99. To separate a number of objects into groups, each having the same number.

Study Recitation

What number of ones are there in each line?



How many groups are there in each line? What number of ones are there in each group?

The number of ones in each line is divided into two equal parts. Each of the two equal parts of any number is *one half* (also written $\frac{1}{2}$) the number. Two halves of any number together equal the whole of it.

Practice

1. What is $\frac{1}{2}$ of 2 1's? What are $\frac{2}{2}$ of 2 1's? Of 2?
2. What is $\frac{1}{2}$ of 4 1's? What are $\frac{2}{2}$ of 4 1's? Of 4?
3. What is $\frac{1}{2}$ of 6 1's? What are $\frac{2}{2}$ of 6 1's? Of 6?
4. What is $\frac{1}{2}$ of 8 1's? What are $\frac{2}{2}$ of 8 1's? Of 8?
5. What is $\frac{1}{2}$ of 10 1's? What are $\frac{2}{2}$ of 10 1's? Of 10?
6. What is $\frac{1}{2}$ of 12 1's? What are $\frac{2}{2}$ of 12 1's? Of 12?
7. What is $\frac{1}{2}$ of 14 1's? What are $\frac{2}{2}$ of 14 1's? Of 14?

TO THE TEACHER. Have pupils separate different even numbers of objects into two equal parts, and drill them in telling the whole number, the number in each half, the number in two halves. Have pupils put even numbers of dashes, dots, circles, squares, etc., on paper or blackboard, separating each number into two equal groups.

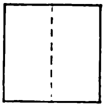
INTEGRAL AND FRACTIONAL UNITS

100. Study Recitation

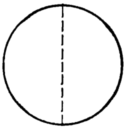
We have found that if a number of objects is separated into two equal parts, each group is $\frac{1}{2}$ of the whole.

If each object is divided into 2 equal parts, each part is $\frac{1}{2}$ of the object.

If the line A————C————B is divided into two equal parts, each of the parts, AC and BC, is $\frac{1}{2}$ of the whole line.



If the square is divided into two rectangles by the dotted line, each rectangle is what part of the square?



If the circle is divided into two equal parts by the dotted line, each part is what part of the circle?

One half of the unit 1 is $\frac{1}{2}$.

– – – – – There are 5 lines in this row; each line is what part of the whole number of lines, or of the 5 lines combined into 1 line ———?

A unit is a single thing, as: 1 line; 1 dot; 1 dollar; 1.

– – – – – 1 of the 5 lines is a unit. If the 5 lines are combined, end to end, so as to make 1 line, as ———, then this line becomes a unit because it is now 1 line instead of 5 separate lines.

If we now separate the line into 5 equal parts, as – – – – –, then each part becomes a unit again, but is a smaller unit than the whole line from which it was taken. Each of these equal units is a fractional part of the large unit, the whole line, and is $\frac{1}{5}$ of it. It is called a **fractional unit.**

2 of the parts are $\frac{2}{5}$ of the whole line, or 2 fractional units, each of which is $\frac{1}{5}$ of the whole line.

4 of the lines are $\frac{4}{5}$ of the whole line, or 4 fractional units, each of which is $\frac{1}{5}$ of the line.

If 4 equal lines, – – – –, are placed in a row, each line is what part of the 4 lines? Why is each line a unit?

If we combine the 4 lines into 1 line ——, that line is a unit. Why? If now we divide it into 4 equal parts, – – – –, each part becomes what fractional unit of the whole line ——?

3 lines, each of the same length as 1 of the fractional units, are $\frac{3}{4}$ of the whole line, or 3 times the fractional unit $\frac{1}{4}$ of the line.

4 lines, each of the same length as 1 of the fractional units, are $\frac{4}{4}$ of the whole line, or 4 times the fractional unit $\frac{1}{4}$ of the line.

If the number 1 is divided into 2 equal parts, each part is a fractional unit, $\frac{1}{2}$. 3 halves means 3 of such fractional units.

If the number 1 is divided into 4 equal parts, each of these parts is the fractional unit, $\frac{1}{4}$. 2 of these parts are 2 of the fractional units, or $\frac{2}{4}$. 3 of these parts are 3 of the fractional units, or $\frac{3}{4}$. 7 such parts are 7 of these fractional units, or $\frac{7}{4}$.

If the number 1 is divided into 5 equal parts, each part is the fractional unit $\frac{1}{5}$. 4 of these parts are 4 of the fractional units, or $\frac{4}{5}$.

When we speak of 7 horses, what is the unit?

When we speak of $\frac{7}{8}$, what is the unit? What kind of a unit? Answer: A fractional unit.

When we speak of 9 dollars, what is the unit?

When we say $\frac{5}{9}$, what is the unit? What kind of a unit? How many of these units are there in $\frac{5}{9}$? How many of these units are there in $\frac{7}{9}$?

When we speak of 2 bushels and 3 bushels, what is the unit in each case? What is the sum of these units? What is their difference?

When we speak of $\frac{2}{7}$ and $\frac{3}{7}$, what is the unit in each case? What is the sum of the units? What is the difference of the units?

If we say 3 desks, what is the unit?

2 times 3 desks are how many desks? How many units are there in this number? What is 1 of the units?

If we say 2 times 3, what is the product? How many units are there in the product? What is each unit?

If we say 2 times $\frac{3}{8}$, how many 8ths are there? What is the unit?

When we speak of a number of things, no matter what kind, 1 thing of that kind is the **unit**. Show what is meant by this statement.

If we speak of a number, as: 4, 5, 7, without giving it any name, what is the unit?

When we write a fraction, as: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{12}$, 1 of the kind named is the unit; as, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{12}$.

TO THE TEACHER. Have pupils draw lines 12 inches long on the board and then divide the lines into equal parts and tell what fraction of the whole line each of the equal parts is; what fraction of the whole line two or more of the parts are; and what fraction one or more of the equal parts is of two or more parts. Question thoroughly to see that pupils understand the nature of the fractional unit and of the fraction.

REVIEW

101. A **fraction** is one or more fractional units: $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{7}{6}$.

A **fractional unit** is made by dividing a unit into equal parts; it is one of the equal parts of a unit: $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{5}$.

A fraction is written with two numbers, one above and the other below a line: $\frac{3}{8}$.

The **denominator** of a fraction is the number below the line, and shows into how many parts the unit is divided. It tells the unit.

If we write the denominator of any fraction with 1 for its numerator, we have the fractional unit of the fraction.

ILLUSTRATION: $\frac{5}{6}$. Writing the denominator 6 with 1 for its numerator thus, $\frac{1}{6}$, we have the fractional unit $\frac{1}{6}$. There are 5 of these fractional units in $\frac{5}{6}$.

The **numerator** is the number above the line, and shows how many fractional units there are in the fraction.

ILLUSTRATION: $\frac{3}{8}$. The numerator shows that three fractional units are taken. The denominator shows that the fractional unit is $\frac{1}{8}$, or that the unit has been divided into 8 equal parts.

The numerator and denominator of a fraction are called the **terms** of a fraction.

What is the fractional unit, and how many fractional units are there in each of the following fractions: $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{7}$?

We have learned in Division that if one number is written above another with a line between them, the upper number is to be divided by the lower; thus, $\frac{8}{2}$ denotes that 8 is to be divided by 2, or that $\frac{1}{2}$ of 8 is to be taken. $\frac{8}{2} = 8 \div 2 = \frac{1}{2}$ of 8.

$\frac{8}{2}$ denotes also 8 fractional units, each of which is $\frac{1}{2}$.

If 12 is written over 3, as $\frac{12}{3}$, it denotes that 12 is to be divided by 3, or that $\frac{1}{3}$ of 12 is to be found. $\frac{12}{3} = 12 \div 3 = \frac{1}{3}$ of 12.

$\frac{12}{3}$ denotes 12 fractional units, each of which is $\frac{1}{3}$.

What does $\frac{16}{4}$ denote?

What does $\frac{25}{5}$ denote?

If 5 is written over 3, as $\frac{5}{3}$, it denotes that 5 is to be divided by 3, or that $\frac{1}{3}$ of 5 is to be found. $\frac{5}{3} = 5 \div 3 = \frac{1}{3}$ of 5. What does $\frac{9}{5}$ denote?

$\frac{5}{3}$ denotes how many fractional units? What is each fractional unit?

If 7 is written over 4, as $\frac{7}{4}$, it denotes that 7 is to be divided by 4, or that $\frac{1}{4}$ of 7 is to be found. $\frac{7}{4} = 7 \div 4 = \frac{1}{4}$ of 7. What does $\frac{19}{6}$ denote?

A **fraction** is an indicated division in which the numerator is the dividend and the denominator the divisor.

FRACTIONAL PARTS OF CONCRETE NUMBERS

102. **Study Recitation**

1. If 2 apples are to be divided equally among 3 boys, each apple is divided into 3 parts, and of 1 apple each boy will have 1 part, or $\frac{1}{3}$ of each apple. Each boy will then have $\frac{2}{3}$ of an apple.

2. If 8 acres of land are to be divided among 4 men, how many acres will each man have? Each will have $\frac{1}{4}$ of 8 acres, or 2 acres; or, 8 acres divided by 4, or $\frac{8}{4}$ acres.

3. If 3 acres are to be divided among 4 men, how many acres will each have? Each will have $\frac{1}{4}$ of 3 acres, or $\frac{3}{4}$ acres, or 3 acres divided by 4.

To show this, draw 3 squares of the same size, each representing 1 acre. Divide the first square into 4 equal parts, 1 part for each man. How much land is there in each division? In the same way divide the other 2 squares. Mark on them the size of each division.

Mark the part of each acre that the first man will have as B; the part of each acre that the second man will have as C; the part of each acre that the third man will have as D; the part of each acre that the fourth man will have as E.

How many parts has each man? Put the 3 squares side by side and mark off the part each of the 4 men will have in one piece. What part of an acre does each have?

REDUCTION OF FRACTIONS

103. To reduce a fraction to a fraction of a higher given denomination.

To THE TEACHER. The following summary of things to be known and done, and the application, are given to guide the teacher in teaching pupils to know and to do what is necessary to perform the required arithmetical operation.

Teach the three things that are to be known by placing examples on the board, thus: reduce. $\frac{4}{5}$ to 10ths; reduce $\frac{3}{4}$ to 8ths; and teach pupils to recognize and name the given fraction, its numerator and denominator, and the denominator of the required fraction.

Then give pupils the first step in the rule and its application in reducing $\frac{3}{8}$ to 16ths. Have pupils learn this step so that they can state it readily and accurately.

Follow in the same way with the second and third steps and their application. Require pupils to reduce other fractions to higher denominations, stating what is done at each step.

A rule is an orderly and concise statement of the necessary steps in an arithmetical process or operation. When the directions given in a rule are studied and applied, under the immediate guidance of the teacher, as above suggested, the rule is understood, and its language becomes a part of the pupils' vocabulary, an important thing if they are to be trained in correct arithmetical expression.

What must be known and done.

To be known:

1. The denominator of the required fraction.
2. The denominator of the given fraction.
3. The numerator of the given fraction.

To be done:

RULE: 1. *Divide the denominator of the required fraction by the denominator of the given fraction.*

2. *Multiply the quotient by the numerator of the given fraction.*

3. *Write the product over the denominator of the required fraction.*

Application of the above to the reduction of $\frac{3}{8}$ to 16ths.

To be known:

1. That 16 is the denominator of the required fraction.
2. That 8 is the denominator of the given fraction.
3. That 3 is the numerator of the given fraction.

To be done:

1. 16 must be divided by 8. Quotient is 2.
2. 2, the quotient, must be multiplied by 3. Product is 6.
3. 6, the product, must be written over 16: $\frac{6}{16}$.

Test of pupils' knowledge of rule and power to apply it in reducing $\frac{4}{5}$ to 10ths, etc.

What is 10? The denominator of the required fraction. What is 5? The denominator of the given fraction. What is 4? The numerator of the given fraction,

What is the first *direction* in the rule? Apply it.
What is the second direction in the rule? Apply it.
What is the third direction in the rule? Apply it.
Apply the test in reducing $\frac{5}{7}$ to 14ths; $\frac{5}{8}$ to 24ths; $\frac{4}{9}$ to 18ths.

1. State the following and give the rule:

To reduce a fraction to a fraction of a higher denomination, —.

Short process in work of reduction:

2. Reduce $\frac{7}{9}$ to 27ths.

9 in 27, 3 times. 7 times 3 = 21. $\frac{21}{27}$.

104. ORAL EXERCISE

TO THE TEACHER. Have pupils do work silently with books before them, giving orally the results, thus: "Three-fourths equal (here the operation is performed mentally) six-eighths." "Two-ninths equal —— four-eighteenths." For seat work, pupils write the required fractions.

1. Change $\frac{3}{4}$ to 8ths; $\frac{2}{9}$ to 18ths; $\frac{3}{7}$ to 21sts.

2. Change $\frac{4}{5}$ to fractions where denominators are $\frac{}{10}, \frac{}{15}, \frac{}{25}$.

3. Change $\frac{3}{4}$ to fractions having the following denominators: $\frac{}{8}, \frac{}{12}, \frac{}{16}, \frac{}{20}$. Change the resulting fractions back to $\frac{3}{4}$. What is the process?

4. Change $\frac{2}{3}$ to fractions having the following denominators: $\frac{}{6}, \frac{}{9}, \frac{}{12}, \frac{}{15}$. Change the resulting fractions back to $\frac{2}{3}$. What is the process?

5. Change $\frac{3}{5}$ to fractions having the following denominators: $\frac{}{20}, \frac{}{10}, \frac{}{25}, \frac{}{15}$.

6. Change $\frac{2}{7}$ to fractions having the following denominators: $\frac{}{14}, \frac{}{35}, \frac{}{49}, \frac{}{21}$.

7. Change $\frac{5}{6}$ to fractions having the following denominators: $\frac{}{12}, \frac{}{36}, \frac{}{24}, \frac{}{18}$.

8. Change $\frac{7}{8}$ to fractions having the following denominators: $\frac{}{40}, \frac{}{16}, \frac{}{32}, \frac{}{56}$.

9. Change $\frac{5}{9}$ to fractions having the following denominators: $\frac{}{45}, \frac{}{18}, \frac{}{96}, \frac{}{54}$.

10. Change $\frac{7}{10}$ to fractions having the following denominators: $\frac{}{20}, \frac{}{90}, \frac{}{40}, \frac{}{30}$.

11. Change $\frac{5}{12}$ to fractions having the following denominators: $\frac{}{36}, \frac{}{48}, \frac{}{24}, \frac{}{60}$.

Written Exercises

12. Reduce $\frac{5}{12}$ to fractions having the following denominators: $\frac{}{252}, \frac{}{384}, \frac{}{528}$.

13. Reduce $\frac{13}{14}$ to fractions having the following denominators: $\frac{}{112}$, $\frac{}{168}$, $\frac{}{280}$.

14. Reduce $\frac{5}{27}$ to fractions having the following denominators: $\frac{}{162}$, $\frac{}{216}$, $\frac{}{405}$.

15. Analyze each of the foregoing examples, from No. 3 to No. 11.

ILLUSTRATION: Change $\frac{3}{4}$ to 12ths. ANALYSIS: $1 = \frac{12}{12}$. $\frac{1}{4} = \frac{1}{4}$ of $\frac{12}{12} = \frac{3}{12}$. $\frac{3}{4} = 3$ times $\frac{3}{12}$ or $\frac{9}{12}$.

16. In reducing the above fractions to fractions of a higher denomination, what has been done in each case?

By what number must you multiply both terms of:

17. $\frac{3}{4}$ to change it to 8ths? 12ths? 16ths? 24ths? 36ths? 48ths?

18. $\frac{4}{7}$ to change it to 14ths? 56ths? 35ths? 63ds? 49ths? 28ths?

19. $\frac{2}{9}$ to change it to 18ths? 27ths? 54ths? 36ths? 45ths? 63ds?

105. Exercises preparatory to reducing fractions to lower and to lowest terms.

By what number must you divide the terms of each of the resulting fractions in example 17 above to change it back to 4ths? In example 18, to change it back to 7ths?

If both terms of a fraction are multiplied by the same number, it is reduced to **higher terms**.

If both terms of a fraction are divided by the same number, it is reduced to **lower terms**.

What numbers will divide each number in each of the following sets:

1. 4
6

2. 6
8

3. 10
12

4. 6
18

5. 8
24

6. 10
12

7. 12
14

8. 14
18

9. 14
16

10. 10
20

11. 12
22

12. 18
24

13. 8
20

14. 9
15

15. 9
21

16. 9
24

17. 10
30

18. 8
18

19. 12
20

20. 7
21

21. 8
32

22. 7
28

23. 16
24

24. 10
25

25. 15
25

26. 15
30

27. 20
30

28. 30
40

106. Reduction of fractions to lower terms.

Write the numbers given above, placing a line between the numbers given in each set. Reduce the resulting fractions to lower terms by dividing. Give answers without writing them.

Make and give a rule for reducing fractions to lower terms.

107. Reduction of fractions to lowest terms.

A fraction is in its **lowest terms** when no number except 1 will divide both numerator and denominator.

Reduce the following fractions to their *lowest* terms:

1. $\frac{6}{8}$

2. $\frac{56}{84}$

3. $\frac{32}{36}$

4. $\frac{12}{14}$

5. $\frac{64}{80}$

6. $\frac{32}{40}$

7. $\frac{24}{28}$

8. $\frac{12}{39}$

9. $\frac{35}{40}$

10. $\frac{27}{36}$

11. $\frac{9}{45}$

12. $\frac{27}{30}$

13. $\frac{48}{64}$

14. $\frac{16}{28}$

15. $\frac{18}{20}$

108. Drill in finding the least common multiple of several numbers.

(Preparatory to reducing fractions to other fractions having the least common denominator.)

A **multiple** of a number is a number that contains the other exactly.

A **common multiple** of several numbers is a number that will contain each of the numbers exactly.

18 is a common multiple of 2, 3, 6, and 9.
27 is a common multiple of 3 and 9.
18 is a common multiple of 3 and 9.

The **least common multiple** of several numbers is the smallest common multiple of the numbers.

1. Give the least common multiple of 2 and 3; of 4 and 5; of 2, 4, and 6.

ILLUSTRATION: Although 6, 12, 18, etc. are all common multiples of 2 and 3, the *least* number that will exactly contain 2 and 3 is 6. The least common multiple of 4 and 5 is 20; and the l. c. m. of 2, 4, and 6 is 12.

To find a common multiple of three numbers when it cannot be seen readily, find it first of any two of them; then of that number and the third number.

2. Find a common multiple of 4, 3, and 9.

ILLUSTRATION: A common multiple of 3 and 9 is 9; of 9 and 4 is 36; or, a common multiple of 3 and 4 is 12; of 12 and 9 is 36.

Find the least common multiples of:

3. 2, 3
4. 2, 4
5. 4, 6
6. 2, 8
7. 3, 6
8. 2, 5
9. 6, 12
10. 6, 18

11. 2, 10
12. 3, 10
13. 4, 5
14. 3, 7
15. 4, 9
16. 5, 6
17. 3, 5
18. 4, 10
19. 5, 10
20. 5, 15
21. 3, 12
22. 12, 2
23. 2, 4, 6
24. 3, 4, 2
25. 2, 4, 5
26. 3, 2, 5
27. 2, 6, 5
28. 3, 6, 5
29. 3, 6, 4
30. 2, 4, 8
31. 3, 6, 9
32. 4, 5, 10
33. 2, 8, 16
34. 3, 6, 18
35. 24, 6, 8
36. 32, 4, 8
37. 36, 12, 3
38. 27, 9, 3
39. 18, 2, 6
40. 20, 5, 4
41. 30, 5, 6
42. 15, 5, 3

To the Teacher. In the above examples small numbers are used, and the pupils should be able to tell the least common multiple by inspection, if preceding work in the book has been properly done. If they are not able to find the required multiple by inspection, show them how it may be found by the method ordinarily used with larger numbers. This work is given here to prepare pupils for work in adding and subtracting fractions.

Two or more fractions have a **common denominator** when all the denominators are the same number.

Illustration: $\frac{8}{12}, \frac{7}{12}, \frac{9}{12}$. 12 is the common denominator of the several fractions.

In finding a common denominator of fractions, it is best to find the smallest one, or the least common denominator.

109. To reduce fractions to other fractions having a common denominator.

Rule: 1. *Find a common multiple of the denominators.*

2. *Reduce each fraction to a fraction having the common multiple for a denominator.*

Application of the above in the reduction of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$, to fractions having a common denominator.

1. 12 is a common multiple of the denominators 3, 4, 6.

2. Each of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$ is to be reduced to 12ths. $\frac{2}{3}=\frac{8}{12}$. $\frac{3}{4}=\frac{9}{12}$. $\frac{1}{6}=\frac{2}{12}$.

1. State the following and give the rules:

To reduce fractions to fractions having a common denominator, ——. To fractions having the least common denominator, ——.

2. To reduce a fraction to a fraction of a higher denomination, ——.

Short process in work of reduction.

3. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{8}$, to fractions having a common denominator.

A common multiple of 3, 4, and 8 is 24.

$\frac{2}{3}$ ··· 3 in 24, 8 times. $2 \times 8 = 16$. $\frac{16}{24}$
$\frac{3}{4}$ ··· 4 in 24, 6 times. $3 \times 6 = 18$. $\frac{18}{24}$
$\frac{1}{8}$ ··· 8 in 24, 3 times. $3 \times 1 = 3$. $\frac{3}{24}$

ANALYSIS: $1 = \frac{24}{24}$. $\frac{1}{3} = \frac{8}{24}$. $\frac{2}{3} = 2$ times $\frac{8}{24} = \frac{16}{24}$
$1 = \frac{24}{24}$. $\frac{1}{4} = \frac{6}{24}$. $\frac{3}{4} = 3$ times $\frac{6}{24} = \frac{18}{24}$
$1 = \frac{24}{24}$. $\frac{1}{8} = \frac{3}{24}$.

Reduce the following fractions to equivalent fractions having a common denominator, or the least common denominator. Analyze, and then solve by dividing and multiplying as in example 3:

4. $\frac{1}{2}$, $\frac{3}{4}$
5. $\frac{1}{2}$, $\frac{5}{8}$
6. $\frac{2}{3}$, $\frac{5}{6}$
7. $\frac{1}{2}$, $\frac{2}{5}$

8. $\frac{5}{6}$, $\frac{7}{12}$
9. $\frac{1}{6}$, $\frac{5}{18}$
10. $\frac{1}{2}$, $\frac{7}{10}$
11. $\frac{2}{3}$, $\frac{3}{10}$
12. $\frac{3}{4}$, $\frac{4}{5}$
13. $\frac{2}{3}$, $\frac{4}{7}$
14. $\frac{3}{4}$, $\frac{4}{9}$
15. $\frac{2}{5}$, $\frac{1}{6}$
16. $\frac{2}{3}$, $\frac{4}{5}$
17. $\frac{3}{4}$, $\frac{9}{10}$
18. $\frac{2}{5}$, $\frac{3}{10}$
19. $\frac{1}{5}$, $\frac{7}{15}$
20. $\frac{2}{3}$, $\frac{5}{12}$
21. $\frac{7}{12}$, $\frac{1}{2}$
22. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{6}$
23. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$
24. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$
25. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{4}{5}$
26. $\frac{1}{2}$, $\frac{2}{6}$, $\frac{3}{5}$
27. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{4}{5}$
28. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{2}{9}$
29. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{7}{10}$
30. $\frac{7}{30}$, $\frac{3}{5}$, $\frac{5}{6}$
31. $\frac{5}{18}$, $\frac{1}{2}$, $\frac{2}{6}$
32. $\frac{3}{4}$, $\frac{7}{32}$, $\frac{5}{8}$
33. $\frac{5}{24}$, $\frac{4}{6}$, $\frac{3}{8}$
34. $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{16}$
35. $\frac{5}{12}$, $\frac{7}{36}$, $\frac{2}{3}$

110. To change a whole or a mixed number to an improper fraction.

A **mixed number** is a whole number and a fraction combined. $4\frac{1}{2}$, $3\frac{3}{7}$, $2\frac{3}{4}$, $156\frac{2}{7}$, $300\frac{5}{8}$, are mixed numbers.

A **proper fraction** is a fraction whose numerator is less than its denominator. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{2}{9}$, $\frac{11}{15}$, are proper fractions.

An **improper fraction** is one whose numerator is equal to or greater than its denominator. $\frac{4}{4}$, $\frac{7}{6}$, $\frac{9}{8}$, $\frac{12}{3}$, are improper fractions.

111. To reduce a whole number to a fraction.

Study Recitation

1. Change 3 to 8ths. Analysis: $1 = \frac{8}{8}$; $3 = 3$ times $\frac{8}{8}$, or $\frac{24}{8}$.

$$\begin{array}{r} 8 \\ \underline{3} \\ 24 \end{array} \qquad \frac{24}{8}$$

Multiply the required denominator, 8, by the whole number, 3, and write the product, 24, over 8: $\frac{24}{8}$. To do the work without writing it, think 3 times 8, over 8, or $\frac{24}{8}$.

Rule: *Multiply the denominator of the required fraction by the whole number, and write the product over the denominator.*

2. Change the following numbers to 7ths: 2, 3, 5, 8, 7. Change the same numbers to 12ths; to 5ths.

112. To reduce a mixed number to an improper fraction.

Rule: *Multiply the denominator of the fractional part by the whole number. Add the numerator to the product. Write the sum over the denominator.*

To the Teacher. Test pupils' power to apply the rule in reducing $5\frac{2}{7}$ and $4\frac{3}{5}$ to improper fractions.

What is the first direction in the rule? What is the result? What is the second direction in the rule? What is the result? How is it written? What is the result? Give the rule.

Reduce the following mixed numbers to improper fractions:

1. $5\frac{3}{4}$ $3\frac{7}{8}$ $6\frac{1}{5}$ $7\frac{1}{2}$ $5\frac{2}{3}$ $4\frac{5}{9}$
2. $7\frac{1}{2}$ $8\frac{3}{5}$ $5\frac{2}{7}$ $3\frac{7}{8}$ $4\frac{2}{9}$ $20\frac{3}{8}$
3. $12\frac{1}{3}$ $13\frac{4}{5}$ $5\frac{7}{12}$ $15\frac{3}{5}$ $16\frac{2}{3}$ $30\frac{2}{5}$
4. $7\frac{5}{8}$ $8\frac{4}{7}$ $9\frac{5}{6}$ $7\frac{4}{9}$ $12\frac{3}{8}$ $50\frac{3}{4}$

5. Reduce $5\frac{2}{7}$ to an improper fraction.

ANALYSIS: $1 = \frac{7}{7}$. $5 = \frac{35}{7}$. $\frac{35}{7} + \frac{2}{7} = \frac{37}{7}$.

Analyze a number of the above examples.

What is a whole number? What is a fraction? What is a fractional unit? What is an improper fraction? What is a mixed number?

Define numerator, denominator, multiple, common multiple, least common multiple, common denominator, least common denominator.

113. To reduce improper fractions to mixed numbers.

1. Reduce $7\frac{1}{2}$ to halves. What is $7\frac{1}{2}$? What is $\frac{15}{2}$? Reduce $\frac{15}{2}$ to a mixed number.

2. Reduce $5\frac{3}{8}$ to an improper fraction. Reduce the improper fraction back to a mixed number.

3. Make and learn a rule for reducing an improper fraction to a mixed number.

Reduce the following improper fractions to whole or mixed numbers:

4. $\frac{15}{4}$
5. $\frac{27}{8}$
6. $\frac{52}{8}$
7. $\frac{28}{8}$
8. $\frac{17}{8}$
9. $\frac{46}{12}$
10. $\frac{96}{12}$
11. $\frac{19}{4}$
12. $\frac{19}{6}$
13. $\frac{29}{7}$
14. $\frac{82}{10}$
15. $\frac{18}{7}$
16. $\frac{25}{3}$
17. $\frac{45}{5}$
18. $\frac{75}{11}$
19. $\frac{21}{6}$
20. $\frac{48}{9}$
21. $\frac{59}{6}$
22. $\frac{66}{12}$
23. $\frac{22}{7}$
24. $\frac{63}{9}$
25. $\frac{85}{12}$
26. $\frac{84}{11}$
27. $\frac{36}{12}$

28. Give rule for reducing fractions to lowest terms; to higher terms; to other fractions having a common denominator; for reducing mixed numbers to improper fractions; improper fractions to mixed numbers.

ADDITION AND SUBTRACTION OF FRACTIONS

114. Fractions must have the same denominators before they can be added or subtracted. If they have not the same denominators, they must be changed to fractions having the same denominators. It is best to have this denominator the smallest that can be used.

$$\frac{3}{7}+\frac{2}{7}=\frac{5}{7} \qquad \frac{4}{9}+\frac{2}{9}+\frac{1}{9}=\frac{7}{9} \qquad \frac{5}{8}-\frac{3}{8}=\frac{2}{8}$$

Study Recitation

1. Add $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{10}$

$$\frac{2}{3}+\frac{4}{5}+\frac{3}{10}=$$
$$\frac{20}{30}+\frac{24}{30}+\frac{9}{30}=\frac{53}{30}=1\frac{23}{30}$$

The smallest or least common denominator is 30. Reduce each fraction to 30ths. The sum of the numerators $=\frac{53}{30}=1\frac{23}{30}$.

2. From $\frac{7}{8}$ subtract $\frac{5}{6}$

$$\frac{7}{8}-\frac{5}{6}=$$
$$\frac{21}{24}-\frac{20}{24}=\frac{1}{24}$$

The least common denominator is 24. Reduce each fraction to 24ths. Subtract.

3. $\frac{2}{3}+\frac{4}{5} = ?$
4. $\frac{3}{8}+\frac{2}{3} = ?$
5. $\frac{5}{12}+\frac{3}{4} = ?$
6. $\frac{3}{10}+\frac{4}{5} = ?$
7. $\frac{5}{8}+\frac{5}{6} = ?$
8. $\frac{2}{3}+\frac{3}{4}+\frac{1}{2} = ?$
9. $\frac{5}{7}+\frac{1}{2}+\frac{3}{4} = ?$
10. $\frac{3}{8}+\frac{2}{5}+\frac{1}{4} = ?$
11. $\frac{5}{14}+\frac{2}{7}+\frac{1}{2} = ?$
12. $\frac{7}{8}-\frac{2}{3} = ?$
13. $\frac{5}{16}-\frac{2}{8} = ?$
14. $\frac{9}{24}-\frac{1}{4} = ?$
15. $\frac{7}{9}-\frac{3}{5} = ?$
16. $\frac{8}{15}-\frac{2}{5} = ?$
17. $\frac{9}{12}-\frac{1}{2} = ?$
18. $\frac{25}{36}-\frac{5}{12} = ?$
19. $\frac{17}{30}-\frac{1}{3} = ?$
20. $\frac{9}{14}-\frac{2}{7} = ?$

115. To add mixed numbers.

Study Recitation

1. Add $4\frac{1}{2}$ and $3\frac{2}{3}$.

$$\begin{array}{rl} 4\frac{1}{2} & \frac{1}{2}=\frac{3}{6} \\ +\,3\frac{2}{3} & \underline{\frac{2}{3}=\frac{4}{6}} \\ \hline 7 & \\ 1\frac{1}{6} & \quad =\frac{7}{6}=1\frac{1}{6} \\ \hline 8\frac{1}{6} & \end{array}$$

Add the whole numbers 3 and 4. Add the fractions. $\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$. Reduce $\frac{7}{6}$ to a mixed number, $1\frac{1}{6}$. Add to 7. $8\frac{1}{6}$.

2. $5\frac{1}{2}+9\frac{3}{4}=?$
3. $8\frac{2}{3}+1\frac{5}{6}=?$
4. $7\frac{3}{5}+12\frac{7}{10}=?$
5. $8\frac{4}{7}+19\frac{1}{3}=?$
6. $14\frac{3}{8}+12\frac{2}{3}=?$
7. $2\frac{1}{2}+4\frac{8}{9}=?$
8. $5\frac{3}{9}+12\frac{4}{5}=?$
9. $16\frac{7}{8}+12\frac{3}{4}=?$
10. $16\frac{7}{9}+10\frac{2}{3}=?$
11. $15\frac{3}{8}+7\frac{5}{6}=?$

12. $8\frac{1}{2}+9\frac{2}{3}+4\frac{1}{4}=?$
13. $13\frac{3}{4}+7\frac{1}{5}+8\frac{1}{2}=?$
14. $21\frac{1}{2}+3\frac{5}{6}+4\frac{2}{3}=?$
15. $12\frac{3}{12}+5\frac{3}{4}+2\frac{1}{2}=?$
16. $10\frac{3}{10}+5\frac{4}{5}+2\frac{1}{2}=?$

Operation

$$\begin{array}{rl} 8\frac{1}{2} & \frac{1}{2}=\frac{6}{12} \\ +\,9\frac{2}{3} & \frac{2}{3}=\frac{8}{12} \\ +\,4\frac{1}{4} & \underline{\frac{1}{4}=\frac{3}{12}} \\ \hline 21 & \qquad \frac{17}{12}=1\frac{5}{12} \\ 1\frac{5}{12} & \\ \hline 22\frac{5}{12} & \end{array}$$

116. To find the difference between mixed numbers.

Study Recitation

1. From $7\frac{3}{8}$ subtract $4\frac{1}{4}$.

$$\begin{array}{r} 7\frac{3}{8} \\ 4\frac{1}{4} \\ \hline 3\frac{1}{8} \end{array}$$

The fraction in the subtrahend is smaller than the fraction in the minuend.

$\frac{1}{4}=\frac{2}{8}$. $\frac{2}{8}$ from $\frac{3}{8}=\frac{1}{8}$. 4 from $7=3$.

2. From $5\frac{1}{2}$ subtract $3\frac{3}{4}$.

$$\begin{array}{r} 5\frac{1}{2} \\ 3\frac{3}{4} \\ \hline 1\frac{3}{4} \end{array}$$

The fraction in the subtrahend is larger than the fraction in the minuend.

$$\begin{array}{rl} 5\frac{1}{2} = & 4+1+\frac{1}{2} \\ 3\frac{3}{4} = & 3+\frac{3}{4} \\ \cline{2-2} & 1+\frac{1}{4}+\frac{1}{2}=1\frac{3}{4} \end{array}$$

When the operation is understood, the minuend need not be separated as in example above.

3. $\begin{array}{r} 9\frac{4}{5} \\ -6\frac{1}{3} \\ \hline \end{array}$ **4.** $\begin{array}{r} 12\frac{3}{7} \\ -4\frac{1}{8} \\ \hline \end{array}$ **5.** $\begin{array}{r} 18\frac{5}{12} \\ -6\frac{3}{4} \\ \hline \end{array}$

6. $19\frac{5}{8}-7\frac{3}{4}=?$ **8.** $30\frac{3}{7}-15\frac{1}{2}=?$ **10.** $13\frac{5}{6}-4\frac{3}{8}=?$

7. $21\frac{2}{3}-5\frac{4}{5}=?$ **9.** $12\frac{5}{12}-9\frac{7}{8}=?$ **11.** $17\frac{4}{5}-3\frac{2}{3}=?$

PROBLEMS

ORAL AND WRITTEN

117. **1.** I buy sugar for $\frac{3}{4}$ of a dollar, cloth for $\frac{7}{8}$ of a dollar, and ribbon for $\frac{1}{8}$ of a dollar. How much change do I get if I pay with a $2 bill?

2. A boys walks $2\frac{1}{2}$ mi. the first hour, $3\frac{2}{3}$ mi. the second hour, and $2\frac{3}{4}$ mi. the third hour. How many miles does he walk in three hours? If he is going 12 mi. from his starting point, how far has he still to travel?

3. A has $15, B has $\frac{1}{4}$ as much, and C has $\frac{1}{2}$ as much. How many dollars do they all have?

4. From a cask containing 30 gallons of water, $10\frac{1}{4}$ gallons are drawn off at one time, and $8\frac{1}{2}$ gallons at another time. How many gallons are left?

5. A boy studies arithmetic $\frac{1}{2}$ an hour, grammar $\frac{2}{3}$ of an hour, and spends $\frac{3}{4}$ of an hour in recitation in each subject daily. How much time each day does he give to arithmetic and grammar?

6. If you have $\$2\frac{1}{4}$ to spend for books, and spend $\$\frac{3}{4}$, how much money do you have left?

Add. Add the fractions mentally:

7.	8.	9.	10.	11.
$5\frac{1}{3}$	$9\frac{3}{8}$	$27\frac{3}{5}$	$15\frac{7}{12}$	$27\frac{2}{9}$
$12\frac{1}{3}$	$13\frac{1}{3}$	$13\frac{7}{10}$	$28\frac{4}{6}$	$18\frac{5}{18}$
$15\frac{3}{4}$	$15\frac{1}{3}$	$28\frac{1}{2}$	$39\frac{3}{4}$	$23\frac{5}{6}$

PROBLEMS

WRITTEN

118. 1. Find the value of $18\frac{3}{4} + 37\frac{1}{2} + 16\frac{2}{3}$.

2. Add $30\frac{1}{4}$, $267\frac{1}{2}$, and $3230\frac{3}{8}$.

3. Subtract $2\frac{5}{8}$ from 7.

4. Subtract $250\frac{3}{5}$ from 300.

5. Subtract $90\frac{1}{3}$ from $108\frac{1}{2}$.

6. From a cask containing 50 gallons of sirup, a grocer sold one customer $16\frac{1}{2}$ gallons, and another $22\frac{7}{8}$ gallons. How many gallons remained unsold? Solve in two ways.

7. If I do $\frac{3}{5}$ of a piece of work, what part of it remains to be done?

8. A man does $\frac{3}{8}$ of a piece of work on Monday, and $\frac{5}{16}$ of it on Tuesday. He finishes it on Wednesday.

What part of the work does he do on Wednesday? State the first question that should be answered in solving the example.

9. $\frac{5}{12}$ of a man's farm is a wood lot, $\frac{3}{8}$ of it is a grass lot, and the remainder is used for grain. What portion of the land is used for growing grain?

10. There are $18\frac{5}{8}$ lb. of butter in a jar, and the jar weighs $5\frac{3}{4}$ lb. What is the combined weight of the butter and jar?

11. A boy lives $1\frac{1}{2}$ miles from the schoolhouse. How far must he walk in going to school and returning once each school day for a week?

12. From a jar containing 18 lb. of butter, a grocer sells $2\frac{3}{4}$ lb. to one customer and $6\frac{1}{2}$ lb. to another. How many pounds are left in the jar?

13. The length of a rectangular picture frame is $18\frac{3}{4}$ inches; its width is $12\frac{1}{2}$ inches. What is the entire distance around it? Draw the outline of the frame on the board, exact size.

14. From a piece of cloth that contained $25\frac{1}{2}$ yd., there were sold $3\frac{1}{2}$ yd., $7\frac{1}{8}$ yd., and $12\frac{3}{4}$ yd. How many yards remained?

15. The sum of two numbers is 1578, and one of them is 923. What is the other?

The sum of two numbers is $19\frac{2}{3}$, and one of them is $6\frac{5}{8}$. What is the other?

16. Having the sum of two numbers and one of the numbers, how do you find the other number?

17. Draw a line $4\frac{3}{4}$ inches long; extend it $2\frac{5}{8}$ inches. What is the entire length of the line? How much further must it be extended to be 1 foot long?

18. Find products. 278 × 36. 397 × 75.

19. 567 × 82. 709 × 89. 457 × 81.

20. Prove that your products are correct by dividing each by the multiplier used to produce it.

21. Find the sum of the products in example 19.

22. Give rapidly the product of each of the following numbers by 9: 12, 9, 7, 11, 8, 6, 4, 10, 5, 3, 2.

23. Give products of the same numbers by 12; by 10; by 8; by 6; by 4; by 7; by 3; by 2.

24. Give orally products of the following numbers by 2, 3, 4, 5: 10, 20, 15, 18, 16, 25, 17, 30, 19, 35, 40, 45, 50.

25. Give orally quotients of the following numbers by 2, 3, 4: 24, 36, 48, 60, 72, 84, 96, 108, 120, 144.

26. Give quotients of the following numbers, divided by 5, 10, 11: 30, 45, 55, 70, 85, 77, 37, 57, 89, 76.

PART III

FACTORING

119. A **prime number** is a whole number not exactly divisible by any other whole number except itself and 1. 2, 3, 5, 7, 11, 13, 17, 19, are prime numbers. Why?

The prime numbers below 20 should be learned.

A **factor** of a number is one of the whole numbers, which, multiplied together, produce the number.

$3 \times 4 = 12$. 3 and 4 are factors of 12. 2, 2, 3, are the **prime factors** of 12, because they are *prime numbers* whose product is 12.

Learn:

The prime factors of 4 are 2 and 2, or two 2's.

The prime factors of 6 are 2 and 3.

The prime factors of 8 are 2 and 2 and 2, or three 2's.

The prime factors of 9 are 3 and 3, or two 3's.

The prime factors of 10 are 2 and 5.

The prime factors of 12 are 2 and 2 and 3, or two 2's and 3.

To THE TEACHER. Each of the following numbers is the product of two factors which should have been learned in the study of the multiplication table. If any pupil cannot give these factors of any of the numbers instantly, he should study the multiplication table until he can do so.

None of these factors is more than 12. If the prime factors of the composite numbers from 4 to 12 inclusive have been thoroughly learned, then to

give the prime factors of any of the following numbers requires two acts of the memory:

(1) Memory of the two factors of the number as given in the multiplication table.

(2) Memory of the prime factors of each of these numbers.

Give the two factors most nearly equal of each of the following numbers. Give the prime factors of each of these factors in the order of their size, the smallest first.

ILLUSTRATION: 14 = 2 × 7. 15 = 3 × 5. 24 = 4 × 6. (There are other sets of two factors of 24: 24 = 3 × 8; 24 = 2 × 12. The first set should be given because the two factors *most nearly equal* are called for.) 20 = 4 × 5. The prime factors of 4 are 2 and 2. 5 is a prime number. The prime factors of 20 are 2, 2, and 5.

ORAL STATEMENT. "The factors of 28 are 4 and 7. The prime factors of 4 are 2 and 2. 7 is a prime number. The prime factors of 28 are 2, 2, and 7."

CONDENSED ORAL STATEMENT. (Think the factors of 28, 4 and 7; think the prime factors of 4 and 7.) State, "The prime factors of 28 are 2, 2, and 7."

Find the prime factors of:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
10	21	32	45	60	80	100
12	22	33	48	63	81	108
14	24	35	49	64	84	110
15	25	36	50	66	88	120
16	27	40	54	70	90	121
18	28	42	55	72	96	132
20	30	44	56	77	99	144

EXACT DIVISORS

120. An **even number** is a number whose unit figure is 0, 2, 4, 6, 8.

2 is an exact divisor of all even numbers. 48, 56, 34, are even numbers. Why?

3 is an **exact divisor** of a number when it will exactly divide the sum of its digits.

ILLUSTRATIONS: In 69 the sum of the digits 6 and 9 is 15. 3 divides 15. 3 will divide 69.

In 351 the sum of the digits 3, 5, and 1 is 9. 3 divides 9. 3 will divide 351.

4 is an exact divisor of any number when it will divide the number represented by its tens and units figures, or when the tens and units figures are 0's.

ILLUSTRATION: In the number 648, 48, the number represented by the tens and units figures, is divisible by 4. 648 is divisible by 4. In the number 600 the tens and units figures are zeros. 600 is divisible by 4.

5 is an exact divisor of a number when its unit figure is 5 or 0: 25, 30, 90, 105, 6235 are divisible by 5. Why?

10 is an exact divisor of a number when its unit figure is 0.

Tell which of the following numbers are divisible by 2; by 3; by 4; by 5; by 10. Give quotients. 26, 31, 33, 34, 38, 41, 46, 49, 50, 51, 52, 57, 58, 60, 65, 74, 76, 78, 80, 81, 86, 85, 90, 100.

CANCELLATION

DRILL

121. Name all the divisors of each number, giving the largest divisor first:

4	8	8	16	24	20	30
6	10	12	8	12	10	20
48	32	36	35	45	30	27
24	48	32	25	20	15	18
15	40	50	35	75	18	16
20	30	25	45	50	9	12

Study Recitation

122. Cancellation is the process of striking out the same factor in the dividend and divisor. It is used to shorten the operation where the dividend and divisor can be divided by the same number.

1. Multiply $\frac{6}{4}$ by 8.

$\frac{6}{4} \times 8 = \frac{48}{4} = 12.$ If the numerator and denominator of the fraction are divided by 2, the result is $\frac{3}{2}$. $\frac{3}{2} \times 8 = \frac{24}{2} = 12.$

$\frac{6}{4} \times \frac{8}{1} = \frac{48}{4} = 12.$ Divide the numerator and the denominator of the product by 4, and the result is $\frac{12}{1}$, or 12.

$\frac{6}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{1} = \frac{12}{1} = 12$ If we divide the denominator 4 and the numerator 8 by 4, before multiplying, we have 6×2 left in the numerator, and 1×1 in the denominator. Multiplying 6×2 and 1×1, we have $\frac{12}{1}$, or 12.

2. Multiply $\frac{4}{2}$ by $\frac{10}{8}$ by $\frac{16}{5}$.

$\frac{4}{2} \times \frac{10}{8} \times \frac{16}{5}$ means the same as $4 \times 10 \times 16$, or 640, divided by $2 \times 8 \times 5$, or 80. $640 \div 80 = 8$.

$$\frac{640}{80} = \frac{4 \times 10 \times 16}{2 \times 8 \times 5} \begin{array}{l}\text{Dividend}\\ \text{Divisor}\end{array}$$

If a factor in the dividend and a factor in the divisor are divided by the same number, the quotient is not changed.

Illustration:

$$\frac{\overset{2}{\cancel{4}} \times \overset{2}{\cancel{10}} \times \overset{2}{\cancel{16}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{8}} \times \underset{1}{\cancel{5}}}$$

Divide the factor 10 in the dividend and the factor 5 in the divisor by 5, crossing out the 10 and 5. Write 2 over the 10 and 1 under the 5. Divide the factor 16 in the dividend and the factor 8 in the divisor by 8, crossing out the 16 and 8. Write 2 over the 16 and 1 under the 8.

Divide the factor 4 in the dividend and the factor 2 in the divisor by 2, crossing out the 4 and 2. Write 2 over the 4 and 1 under the 2.

Multiply together the remaining factors in the dividend. $2 \times 2 \times 2 = 8$. Multiply the remaining factors in the divisor, $1 \times 1 \times 1 = 1$. $\frac{8}{1} = 8$.

Solve by cancellation:

3. $\frac{4}{6} \times \frac{8}{10} \times \frac{24}{12} = ?$

4. $\dfrac{32 \times 36 \times 35}{48 \times 20 \times 15} = ?$

5. $\dfrac{27 \times 12 \times 40 \times 50}{18 \times 26 \times 28 \times 24} = ?$

6. $\dfrac{22 \times 11 \times 55 \times 16}{20 \times 45 \times 25 \times 12} = ?$

7. $\dfrac{80 \times 15 \times 12 \times 20}{60 \times 50 \times 48 \times 32} = ?$

8. $\dfrac{25 \times 16 \times 9 \times 10}{10 \times 3 \times 4 \times 5} = ?$

9. A farmer exchanges 8 dozen eggs worth 12¢ a dozen for sugar worth 6¢ a pound. How many pounds does he get?

SUGGESTION: $\frac{8 \times 12}{6}$. Cancel.

Solve:

10. $\frac{3}{5} \times \frac{5}{6} \times \frac{8}{12} = ?$

11. $\frac{7}{8} \times \frac{9}{14} \times \frac{5}{12} = ?$

12. $4\frac{1}{2} \times \frac{8}{10} \times \frac{1}{8} = ?$

13. $4\frac{2}{3} \times 6\frac{6}{9} = ?$

14. $24\frac{1}{2} \times 2\frac{2}{7} = ?$

15. $15\frac{1}{4} \times 2\frac{1}{6} = ?$

16. If 8 barrels of apples are worth $18, how much is one barrel worth?

Indicate the operation without performing it, as, $\$\frac{18}{8}$. This is the expression for the cost of one barrel.

$5 \times \$\frac{18}{8}$ is the expression for the cost of 5 barrels.

$6 \times \$\frac{18}{8}$ is the expression for the cost of 6 barrels.

$2 \times \$\frac{18}{8}$ is the expression for the cost of 2 barrels.

Cancel in each case, and find the cost of 5 barrels; of 6 barrels; of 2 barrels.

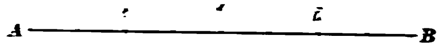
17. A farmer sells 5 head of cattle for $156. What is the expression for the cost of one head? Write it in the form of a fraction. What will be the expression for the cost of 3 head? What is the cost of 3 head?

18. A farmer sells 16 barrels of potatoes, each holding $2\frac{1}{2}$ bushels, at $\frac{3}{4}$ of a dollar per bushel. What is the expression for the amount he receives? How much does he receive?

MULTIPLICATION OF FRACTIONS

Study Exercise

123. The line *AB* is divided into 8 equal parts, each of which is $\frac{1}{8}$ of the whole line. The line *Ac* is $\frac{2}{8}$ of the whole line: it is also $\frac{1}{4}$ of the whole line. The line



Ae is $\frac{4}{8}$ of the whole line: it is also $\frac{2}{4}$, also $\frac{1}{2}$, of the whole line. The line *Af* is $\frac{6}{8}$ of the whole line: it is also $\frac{3}{4}$ of the whole line. In each of the fractions $\frac{1}{8}$, $\frac{2}{8}$, $\frac{4}{8}$, $\frac{6}{8}$, the denominator shows the number of parts into which the line has been divided or the fractional unit, and the numerator shows the number of the parts taken or the number of the fractional units in each.

2 times $\frac{1}{8}$ of the line equals $\frac{2}{8}$ or $\frac{1}{4}$ of the line, or the line *Ac*. 4 times $\frac{1}{8}$ of the line equals $\frac{4}{8}$ or $\frac{1}{2}$ of the line, or the line *Ae*. Multiplying the numerator of the fraction $\frac{1}{8}$ by 2, multiplies the fraction by 2, because it multiplies the number of parts or the number of fractional units by 2 and gives $\frac{2}{8}$.

Multiplying the numerator of the fraction $\frac{1}{8}$ by 4, multiplies the fraction by 4, because it multiplies the number of parts or the number of fractional units by 4, and gives $\frac{4}{8}$. Dividing the denominator of the fraction $\frac{1}{8}$ by 2, multiplies the fraction by 2, because it makes each part, or fractional unit, $\frac{1}{4}$; $\frac{1}{4}$ is 2 times as large as $\frac{1}{8}$.

Dividing the denominator of the fraction $\frac{1}{8}$ by 4, multiplies the fraction by 4, because it makes each part or fractional unit $\frac{1}{2}$, or 4 times as large as $\frac{1}{8}$.

Multiplying the numerator or dividing the denominator of a fraction by a whole number, multiplies the fraction by that number.

Draw a line and divide it into 12 equal parts.

Each part is what fraction of the whole line?

Show that 3 times $\frac{1}{12}$ of the line equals $\frac{3}{12}$ of the line.

What is done to change $\frac{1}{12}$ to $\frac{3}{12}$?

Show that 3 times $\frac{1}{12}$ of the line equals $\frac{1}{4}$ of the line.

What is done to change $\frac{1}{12}$ to $\frac{1}{4}$?

In what ways can $\frac{1}{12}$ be multiplied by 3?

In what ways can $\frac{1}{12}$ be multipled by 6? Show by the divisions of the line.

Find product by multiplying numerators:

1. $3 \times \frac{1}{8} = ?$ $\quad 3 \times \frac{1}{5} = ?$ $\quad 3 \times \frac{1}{7} = ?$ $\quad 3 \times \frac{1}{10} = ?$
2. $2 \times \frac{3}{7} = ?$ $\quad 3 \times \frac{3}{10} = ?$ $\quad 4 \times \frac{2}{11} = ?$ $\quad 4 \times \frac{2}{9} = ?$

Find products and reduce to lowest terms:

3. $3 \times \frac{2}{10} = \frac{6}{10} = \frac{3}{5}$ $\quad 3 \times \frac{4}{6} = ?$ $\quad 4 \times \frac{3}{14} = ?$ $\quad 5 \times \frac{2}{12} = ?$

Find products by canceling and multiplying:

4. $4 \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}}} = \frac{4}{3} = 1\frac{1}{3}$ $\quad 3 \times \frac{2}{10} = ?$ $\quad 3 \times \frac{5}{10} = ?$. $\quad 5 \times \frac{4}{12} = ?$

Find products by dividing the denominators:

5. $3 \times \frac{1}{6} = ?$ $\quad 4 \times \frac{1}{8} = ?$ $\quad 5 \times \frac{1}{10} = ?$ $\quad 2 \times \frac{1}{6} = ?$
6. $3 \times \frac{2}{9} = ?$ $\quad 2 \times \frac{5}{12} = ?$ $\quad 4 \times \frac{3}{16} = ?$ $\quad 5 \times \frac{3}{20} = ?$

Find products by canceling and multiplying:

7. $\overset{1}{\cancel{4}} \times \frac{3}{\underset{2}{\cancel{8}}} = 1\frac{1}{2}$ $\quad 6 \times \frac{5}{12} = ?$ $\quad 3 \times \frac{5}{9} = ?$ $\quad 2 \times \frac{7}{16} = ?$

To THE TEACHER. Every example in multiplication, when either factor, or neither factor, is a whole number, may be treated as a case of multiplying one fraction by another.

To avoid confusion in the pupil's mind, but one rule is given. When the application of this rule is thoroughly mastered, pupils may very readily be led to see that it involves the processes usually taught for the different cases, and they may be employed if the teacher so desires.

Whole numbers and mixed numbers may always be written or thought of as improper fractions. $4 = \frac{4}{1}$. $3\frac{5}{8} = \frac{29}{8}$.

124. To multiply, when one factor is a fraction, a whole number, or a mixed number.

When both are fractions, or mixed numbers.

RULE: 1. *Write or think of both multiplier and multiplicand as fractions, with the sign of multiplication between them.*

2. *Cancel factors common to numerator and denominator.*

3. *Multiply together the remaining factors of the numerator, and write or think the product over the product of the remaining factors of the denominator.*

4. *If the product is an improper fraction, reduce it to a mixed number.*

To THE TEACHER. Test pupils' knowledge of the rule and power to apply it by writing each of the following examples on the board and asking the questions given below. When a question calls for a part of the rule as an answer, require it to be learned, given from memory, and applied. Work with pupils until each step in the rule, in order, is thoroughly understood and they are able to apply it and state it.

Study Recitation

1. $2\frac{1}{2} \times 3 = 7\frac{1}{2}$
2. $\frac{7}{8} \times \frac{12}{14} = \frac{3}{4}$
3. $3\frac{1}{2} \times \frac{4}{5} = 2\frac{4}{5}$
4. $2\frac{4}{5} \times 2\frac{1}{7} = 6$

What is the first direction in the rule? Do as directed. What is the second direction in the rule? Do as directed. What factors of the numerator remain? What factors of the denominator remain?

What is the third direction in the rule? Do as directed. What is the result? What is the fourth direction in the rule? Does it apply? If so, what is the result?

Give the rule.

125. Examples in which one factor is a whole number and the other a fraction.

Write both factors as fractions, cancel when there are common factors in numerator and denominator, and complete the operation.

Think both factors as fractions, cancel, and multiply without any written work. Give results orally.

1. $5 \times \frac{8}{10} = ?$
2. $3 \times \frac{5}{7} = ?$
3. $4 \times \frac{5}{12} = ?$
4. $6 \times \frac{7}{18} = ?$
5. $7 \times \frac{3}{14} = ?$
6. $7 \times \frac{3}{28} = ?$
7. $4 \times \frac{5}{9} = ?$
8. $5 \times \frac{3}{15} = ?$
9. $8 \times \frac{5}{16} = ?$
10. $5 \times \frac{4}{25} = ?$
11. $3 \times \frac{5}{8} = ?$
12. $2 \times \frac{9}{14} = ?$
13. $3 \times \frac{5}{18} = ?$
14. $7 \times \frac{8}{21} = ?$
15. $5 \times \frac{3}{20} = ?$

126. Examples in which one factor is a whole number and the other a mixed number.

Write both factors as fractions and complete the operation:

1. $6 \times 3\frac{1}{2} = ?$
2. $7 \times 4\frac{1}{3} = ?$
3. $5 \times 2\frac{3}{8} = ?$
4. $4 \times 3\frac{7}{8} = ?$
5. $8 \times 2\frac{1}{16} = ?$
6. $8 \times 3\frac{2}{3} = ?$
7. $12 \times 3\frac{3}{4} = ?$
8. $7 \times 8\frac{1}{5} = ?$
9. $7 \times 3\frac{1}{14} = ?$
10. $9 \times 5\frac{5}{18} = ?$
11. $9 \times 2\frac{2}{3} = ?$
12. $8 \times 12\frac{3}{4} = ?$
13. $7 \times 13\frac{3}{5} = ?$
14. $9 \times 13\frac{5}{8} = ?$
15. $8 \times 12\frac{3}{16} = ?$

127. Examples in which both factors are fractions.

Solve them, putting the work on paper or blackboard:

Solve without written work, giving the products orally:

1. $\frac{1}{2} \times \frac{4}{5} = ?$
2. $\frac{7}{8} \times \frac{4}{9} = ?$
3. $\frac{5}{12} \times \frac{2}{3} = ?$
4. $\frac{5}{6} \times \frac{12}{25} = ?$
5. $\frac{12}{15} \times \frac{30}{48} = ?$
6. $\frac{15}{18} \times \frac{24}{30} = ?$
7. $\frac{7}{12} \times \frac{2}{5} = ?$
8. $\frac{8}{15} \times \frac{10}{14} = ?$
9. $\frac{3}{14} \times \frac{7}{12} = ?$

TO THE TEACHER. The following drill exercise is an excellent one in training pupils to attend closely, to remember conditions, and to work rapidly.

DRILL EXERCISE FOR RAPID WORK

Pupils take pencil and paper and place the numbers 1 to 5 in a column. Read the first example above to them, to be solved mentally. At the end of five seconds call "time." At this signal each pupil having the product is to write it as answer to 1. If any pupil does not have the product, he is to place a —— at the right of the 1. Continue with succeeding examples in the same way. When five examples have been solved, have one pupil read his answers and all pupils mark errors.

Practice for the greatest possible speed and accuracy.

Similar work may be done in solving many of the problems in the lists of Oral Problems throughout the book.

128. Examples in which both factors are mixed numbers.

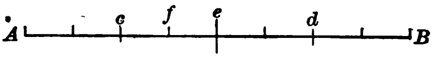
Write both factors as improper fractions, and solve.

1. $2\frac{1}{2} \times 4\frac{2}{5} = ?$
2. $3\frac{1}{8} \times 2\frac{1}{10} = ?$
3. $5\frac{2}{7} \times 4\frac{2}{3} = ?$
4. $27\frac{2}{3} \times 8\frac{2}{5} = ?$
5. $15\frac{1}{2} \times 8\frac{2}{5} = ?$
6. $4\frac{3}{8} \times 2\frac{2}{3} = ?$
7. $8\frac{1}{2} \times 3\frac{1}{9} = ?$
8. $6\frac{2}{3} \times 1\frac{3}{4} = ?$
9. $12\frac{1}{2} \times 5\frac{3}{8} = ?$

DIVISION OF FRACTIONS

129. **Study Recitation**

The line *Ad* is $\frac{6}{8}$ of the whole line; it is also $\frac{3}{4}$ of the whole line. If the line *Ad* is divided into 3 equal parts, the parts are the lines *Ac*, *ce*, and *ed*. Each part is $\frac{2}{8}$, or $\frac{1}{4}$, of the whole line. In $\frac{6}{8}$ there are 6 fractional units; the fraction is divided by 3, by dividing 6, the number of fractional units, by 3, thus: $3)\frac{6}{8}=\frac{2}{8}$, in lowest terms $\frac{1}{4}$.



In $\frac{3}{4}$ there are 3 fractional units; the fraction is divided by 3, by dividing 3, the number of fractional units, by 3, thus: $3)\frac{3}{4}=\frac{1}{4}$.

The line *Ad* is $\frac{3}{4}$ of the whole line. If the line *Ad* is divided into 2 equal parts, the parts are the lines *Af* and *fd*. Each part is $\frac{3}{8}$ of the whole line. In $\frac{3}{4}$ there are 3 fractional units; each fractional unit is $\frac{1}{4}$. In this case the fraction $\frac{3}{4}$ is divided by decreasing the size of the fractional units. This is done by multiplying the denominator by 2, thus, $\frac{3}{4}\div 2=\frac{3}{2\times 4}=\frac{3}{8}$.

RULE: *To divide a fraction by a whole number, divide the numerator or multiply the denominator by the whole number.*

Find quotients by dividing numerators:

1. $\frac{6}{7}\div 2=?$ $\frac{8}{9}\div 4=?$ $\frac{12}{13}\div 6=?$ $\frac{10}{17}\div 5=?$
2. $\frac{24}{25}\div 4=?$ $\frac{15}{16}\div 3=?$ $\frac{48}{49}\div 8=?$ $\frac{35}{37}\div 7=?$

Find quotients and reduce to lowest terms:

3. $\frac{8}{12} \div 2 = \frac{4}{12} = \frac{1}{3}$. $\frac{6}{9} \div 2 = ?$ $\frac{10}{15} \div 5 = ?$ $\frac{9}{15} \div 3 = ?$
4. $\frac{20}{25} \div 4 = ?$ $\frac{16}{18} \div 8 = ?$ $\frac{6}{8} \div 3 = ?$ $\frac{8}{10} \div 4 = ?$

Find quotients by multiplying denominators:

5. $\frac{3}{4} \div 2 = ?$ $\frac{5}{6} \div 4 = ?$ $\frac{3}{8} \div 2 = ?$ $\frac{7}{10} \div 3 = ?$
6. $\frac{2}{3} \div 5 = ?$ $\frac{3}{4} \div 7 = ?$ $\frac{5}{7} \div 8 = ?$ $\frac{2}{9} \div 3 = ?$

130. To divide, when either divisor or dividend is a fraction or a mixed number; when both divisor and dividend are fractions or mixed numbers.

RULE

To be done: 1. *Reduce the mixed numbers to improper fractions.*
2. *Invert the terms of the divisor.*
3. *Change the sign of division to the sign of multiplication.*
4. *Apply the rule for multiplication of fractions.*

Application of the rule to the division of $3\frac{3}{8}$ by $\frac{3}{4}$. $3\frac{3}{8}$ reduced to an improper fraction is $\frac{27}{8}$.

The divisor $\frac{3}{4}$ inverted becomes $\frac{4}{3}$.

On changing the sign and writing the inverted divisor after it, the example becomes $\frac{27}{8} \times \frac{4}{3}$.

Applying the rule for multiplication of fractions, we have:
$\frac{27}{8} \times \frac{4}{3} = \frac{9}{2} = 4\frac{1}{2}$, the quotient of $\frac{3}{8} \div \frac{3}{4}$.

Practice

1. Divide $\frac{5}{8}$ by $\frac{5}{9}$.
2. Divide $\frac{2}{11}$ by $\frac{3}{8}$.
3. Divide $8\frac{5}{9}$ by $\frac{7}{18}$.
4. Divide $15\frac{1}{2}$ by $\frac{3}{2}$.
5. Divide $7\frac{2}{3}$ by $2\frac{7}{8}$.
6. $\frac{2}{7} \div \frac{3}{5} = ?$
7. $\frac{7}{7} \div \frac{5}{12} = ?$
8. $12\frac{3}{4} \div \frac{3}{8} = ?$
9. $6\frac{7}{8} \div \frac{5}{16} = ?$
10. $48\frac{7}{5} \div 1\frac{1}{2} = ?$

11. $2\frac{3}{4} \div \frac{5}{6} = ?$ 13. $\frac{20}{25} \div 2\frac{1}{2} = ?$ 15. $8\frac{2}{3} \div 3\frac{1}{3} = ?$

12. $\frac{5}{12} \div \frac{7}{10} = ?$ 14. $12\frac{1}{2} \div \frac{5}{8} = ?$ 16. $\frac{7}{8} \div \frac{5}{6} = ?$

131. PROBLEMS

Oral and Written

1. $\frac{3}{8}$ of a farm is divided equally among three children. What part does each get? If there are 120 acres in the farm, how many acres do the three children get? How many acres does each get?

2. If 2 yards of cloth will make a vest, how many vests will 10 yards make?

How is the problem solved? Answer: Divide the whole number of yards by the number of yards it takes to make 1 vest.

If $\frac{2}{3}$ of a yard of cloth will make a vest, how many vests will $3\frac{1}{3}$ yd. make? How is the problem solved?

3. If 5 men can build $15\frac{1}{2}$ rods of fence in a day, how many rods of fence can one man build in a day? Analyze.

4. If 9 lb. of butter cost \$ $1\frac{1}{2}$, how much will 4 lb. cost? What is the first question to be answered?

5. How many acres are there in 8 village lots, each containing $\frac{3}{5}$ of an acre?

6. How much will 35 barrels of flour cost at \$ $6\frac{1}{2}$ a barrel?

7. How much will 5 acres of land cost at \$ 40 an acre? How is the problem solved?

8. How much will $\frac{7}{8}$ of an acre of land cost at \$ 40 an acre?

9. How many pieces of land, each containing 3 acres, can be sold from a tract containing 15 acres? How is the problem solved?

10. How many village lots, each containing $\frac{3}{8}$ of an acre, can be sold from a tract containing 15 acres?

11. If a family uses $\frac{4}{9}$ of a barrel of flour in a month, how long will $2\frac{1}{3}$ barrels last? Make a similar problem using whole numbers. How is it solved?

12. If a bushel of corn costs $\frac{5}{8}$ of a dollar, how many bushels can be bought for \$ $6\frac{1}{2}$?

13. What is the weight of 48 sacks of salt if each weighs $12\frac{3}{4}$ lb.?

14. If a barrel holds $2\frac{3}{4}$ bushels of apples, how many barrels will hold 132 bushels?

15. If a man's steps are $2\frac{1}{2}$ feet each, how many steps will he take in walking 150 ft.?

16. Two boys paid \$ 24 for a boat. One paid $\frac{3}{5}$ of the cost, and the other $\frac{2}{5}$. How much did each pay?

17. If $\frac{7}{8}$ of a bushel of nuts is placed in each bag, how many bags will be required for 14 bushels?

ANALYSIS: If $\frac{7}{8}$ of a bushel of nuts is placed in each bag, it will require as many bags to hold 14 bushels as $\frac{7}{8}$ is contained times in 14.

18. If one man can build a fence in $9\frac{3}{5}$ days, in what time can 8 men build it? In what time can 3 men build it?

19. If 3 doz. oranges can be bought for ¾ of a dollar, what is the price of half a dozen? What is the first question to be answered?

20. How many pounds of sugar can I buy for $9 if sugar is sold at the rate of 18½ lb. for a dollar?

21. If 2½ bushels of apples cost $5 ($3), how much will 1 bushel cost? How much will 3½ bushels cost? Solve in two ways.

22. At $2 (⅝ of a dollar) per bushel, how many bushels of apples can be bought for $16 ($16½)?

23. The quotient is 4, the divisor is 3. What is the dividend?

24. The quotient is ¾, the divisor is ⅚. What is the dividend?

25. The divisor is ⅞, the quotient is ⅔. What is the dividend? Having a quotient and divisor given, how can we find the dividend?

26. The quotient is 7, the dividend is 28. What is the divisor? Make and solve a similar example.

27. Having the quotient and dividend given, how can we find the divisor? Give an example.

28. What are the factors of a product called? If a product and one of the factors employed in forming it are given, how can we find the other factor?

29. If 18 yards of ribbon are cut into pieces ¾ of a yard long, how many pieces will it make?

30. How many times can I fill a pail holding $1\frac{1}{4}$ gallons from a tank containing 20 gallons?

31. How many strips of carpet $\frac{3}{4}$ of a yard wide will be needed for a room 24 feet wide? How many yards wide is the room?

32. A woman buys a $6\frac{3}{8}$-yd. remnant of silk worth $1.25 a yard for $5.25. How much does she save by buying the remnant?

33. If $17\frac{1}{2}$ pounds of cream produce $4\frac{1}{2}$ pounds of butter fat, how many pounds of butter fat should $26\frac{1}{2}$ pounds of cream produce?

34. If barbed wire weighs $8\frac{1}{4}$ lb. per 100 ft., what is the weight of the wire in a 4-wire fence 40 rd. long?

35. What is the cost of 2 turkeys at $18\frac{1}{2}$¢ a pound and 3 chickens at $12\frac{1}{2}$¢ a pound? One turkey weighs $12\frac{1}{2}$ lb.; the other, $15\frac{3}{4}$ lb. The chickens weigh 4 lb., $5\frac{1}{2}$ lb., and $4\frac{3}{8}$ lb., respectively.

36. Give rule for addition of fractions; for subtraction of fractions; for multiplication of a whole number, a mixed number, or a fraction, by a mixed number or a fraction; for dividing a whole number, a mixed number, or a fraction, by a whole number, a mixed number, or a fraction.

37. Define: multiple; common multiple; least common multiple; common denominator; least common denominator; prime number; factor; even number.

38. What numbers are divisible by 2, 3, 4, 5, 10?

FRACTIONAL PARTS OF NUMBERS

Study Recitation

132. 1. Find $\frac{3}{8}$ of 24. ANALYSIS: $\frac{1}{8}$ of $24 = 3$. $\frac{3}{8}$ of $24 = 3 \times 3$, or 9.

2d analysis: $\frac{3}{8}$ of $24 = \frac{1}{8}$ of 3×24. $3 \times 24 = 72$. $\frac{1}{8}$ of $72 = 9$.
Use the second analysis when the denominator will not exactly divide the number.

ORAL

Analyze the following:

2. $\frac{3}{5}$ of 25
3. $\frac{7}{8}$ of 42
4. $\frac{2}{3}$ of 16
5. $\frac{5}{12}$ of 24
6. $\frac{7}{10}$ of 60
7. $\frac{3}{4}$ of 11
8. $\frac{5}{9}$ of 12
9. $\frac{5}{6}$ of 16
10. $\frac{3}{7}$ of 63
11. $\frac{8}{9}$ of 54
12. $\frac{6}{7}$ of 84
13. $\frac{5}{9}$ of 63
14. $\frac{4}{5}$ of 80
15. $\frac{3}{4}$ of 32
16. $\frac{5}{8}$ of 80

17. Find $\frac{4}{7}$ of 35. OPERATION: $35 \div 7 = 5$. $4 \times 5 = 20$, or $4 \times 35 = 140$. $140 \div 7 = 20$.

133. To find a fractional part of a number.

RULE: *Divide the number by the denominator of the fraction and multiply the quotient by the numerator.*

Or, *Multiply the number by the numerator, and divide the product by the denominator.*

Work the following without analyzing.

ORAL

1. $\frac{2}{7}$ of 14
2. $\frac{3}{5}$ of 20
3. $\frac{4}{7}$ of 63
4. $\frac{6}{7}$ of 49
5. $\frac{4}{11}$ of 121
6. $\frac{7}{8}$ of 88
7. $\frac{3}{8}$ of 15
8. $\frac{4}{5}$ of 12
9. $\frac{3}{7}$ of 15

Written

10. $\frac{5}{9}$ of 75

11. $\frac{8}{11}$ of 96

12. $\frac{5}{13}$ of 250

13. $\frac{3}{17}$ of 120

14. $\frac{6}{7}$ of 27

15. $\frac{5}{12} \times 34$

If the fraction is not in its lowest terms, reduce it before multiplying.

16. $\frac{6}{9}$ of 33

17. $\frac{8}{12}$ of 40

18. $\frac{4}{16}$ of 80

19. $\frac{7}{21}$ of 90

20. $\frac{6}{8}$ of 28

21. $\frac{7}{14}$ of 36

22. $\frac{8}{32}$ of 24

23. $\frac{9}{45}$ of 50

24. $\frac{5}{15}$ of 120

25. $\frac{20}{30}$ of 48

26. $\frac{13}{39}$ of 63

27. $\frac{7}{35}$ of 55

PROBLEMS

134.

Oral

1. John has $25. William has $\frac{4}{5}$ as many. How many has William? How many dollars have both? How many more dollars has John than William?

2. A gardener has a garden containing 48 square rods. He has $\frac{3}{8}$ of it planted with peas, and $\frac{1}{4}$ of it with beans. What part of the garden is planted with peas and beans? How many square yards?

Draw a plan of the field, showing the part planted with peas; with beans.

3. There are 45 pupils in the third grade, and $\frac{7}{9}$ as many in the fourth grade. How many are there in both grades?

4. A cask contains 50 gallons of sirup. If $\frac{2}{5}$ of it is sold for $1 a gallon, how much does it bring?

WRITTEN

5. In an orchard there are 384 trees. $\frac{3}{8}$ of them are apple trees; $\frac{1}{3}$ of them, plum trees; and the rest are pear trees. How many trees of each kind are there?

6. A ship's cargo is valued at $8750. The ship springs a leak, and $\frac{3}{8}$ of the cargo is thrown overboard. What is the loss? What is the value of what remains? What part of the cargo is left?

7. A man sells $\frac{5}{6}$ of 228 cords of wood for $4 per cord. How much does he receive?

8. Give the rule for finding a fractional part of a number.

Illustrate the application of the rule by giving and solving an example.

To find what fractional part one number is of another.

135. Study Recitation

1. 1 is what part of 2? Of 3? Of 4? Of 5? Of 7? Of 15? Of 20?

To express what fractional part 1 is of any number, write 1 as the numerator and the other number as the denominator of the fraction. $\frac{1}{8}$ means $1 \div 8$. $\frac{7}{8}$ means $7 \div 8$. $\frac{8}{4}$ means $8 \div 4$.

2. 2 is what part of 3? Of 5? Of 7? Of 9? Of 4? Of 8? Of 11?

3. How do you express what part 2 is of any number? Give examples.

4. How do you express what part 3 is of any number? 5? Give examples.

136. To express what part any number is of any other number.

Rule: *Write the number expressing the part as the numerator and the other number as the denominator of the fraction.* Or,

Divide the number expressing the part by the other number.

In expressing the fractional part one number is of another, give the fraction in its lowest terms.

1. 4 is what part of 8?

Analysis: 1 is $\frac{1}{8}$ of 8. 4 is $\frac{4}{8}$, or $\frac{1}{2}$, of 8.

Note to Teacher. Require analysis until pupils are familiar with it, then ask for results without analysis.

2. 5 is what part of 10?
3. 6 is what part of 12?
4. 7 is what part of 21?
5. 6 is what part of 8?
6. 9 is what part of 12?
7. 8 is what part of 14?
8. 6 is what part of 15?
9. 10 is what part of 15?
10. 18 is what part of 21?
11. 20 is what part of 24?

PROBLEMS

Oral

137. 1. In the lower half of a window there are 3 panes of glass in a row; there are 4 such rows of panes in the window. What part of the whole number of panes in the window is the number in the lower row? In the two lower rows? Make other questions and answer them.

2. John earns $2 a day for 6 days. Henry earns $1.50 a day for the same time. Henry's earnings are what part of John's? John's earnings are how many times Henry's?

3. A farmer tests 32 kernels of corn to see how many will grow. 8 kernels fail to grow. What part of the whole number of kernels grow?

4. Give the rule for finding what fractional part one number is of another. Give an example and solve it.

5. Give an example of finding a fractional part of a number. Give the rule.

138. To find a number when a fractional part of it is given.

Study Recitation

1. $\frac{1}{3}$ of a number is 4. $\frac{2}{3}$ of the number is 2×4, or 8.

2. 8 is $\frac{2}{3}$ of what number?

In example 1, 8 is found by multiplying 4, or $\frac{1}{3}$ of a number, by 2. If 8 is divided by 2, the quotient, 4, is $\frac{1}{3}$ of the required number. $\frac{3}{3}$ of the number is 3×4 or 12.

3. A number is divided into 5 equal parts; 3 of these parts are 18. What is one of the parts? What is the whole number?

4. If a number is divided into 5 equal parts, each part is what fraction of the number? 18 is $\frac{3}{5}$ of what number?

ANALYSIS: If 18 is $\frac{3}{5}$ of some number, $\frac{1}{5}$ of that number is $\frac{1}{3}$ of 18, or 6. $\frac{5}{5}$ of the number $= 5 \times 6$, or 30.

5. $\frac{3}{4}$ of a number is 21. What is the number?

ANALYSIS BY QUESTION AND ANSWER: If 21 is $\frac{3}{4}$ of a number, what is $\frac{1}{4}$ of the number? Answer: $\frac{1}{3}$ of 21, or 7. If 7 is $\frac{1}{4}$ of a number, what is the number? Answer: 4 × 7, or 28.

TO THE TEACHER. If, in solving this example, pupils are inclined to say, "If 21 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ is $\frac{1}{4}$ of 21," the teacher should give the example without naming the fractional unit, as: "If 21 is 3 of the equal parts of a number, what is 1 of these parts?" When this question is answered, then follow with: "If 7 is $\frac{1}{4}$ of a number, what is the number?" Then have the original example analyzed.

Analyze the following examples:

6. 10 is $\frac{5}{8}$ of what number?

7. 6 is $\frac{3}{5}$ of what number?

8. 15 is $\frac{5}{7}$ of what number?

9. 21 is $\frac{7}{8}$ of what number?

10. If $\frac{7}{9}$ of a number is 14, what is the number?

139. To find any number when a fractional part of it is known.

RULE: *Divide the part of the number given by the numerator of the fraction, and multiply the product by the denominator.*

1. 36 is $\frac{9}{10}$ of what number? $36 \div 9 = 4$. $10 \times 4 = 40$.

In the same way solve the following, giving results orally:

2. 24 is $\frac{8}{9}$ of what number? 32? 56? 64?

3. 28 is $\frac{7}{8}$ of what number? 35? 56? 63?

4. 30 is $\frac{2}{3}$ of what number? 50? 44? 36?

5. 25 is $\frac{5}{6}$ of what number? 55? 75? 45?

To the Teacher. Pupils may be required to prove these results by finding the fractional part of the number found in each case.

In example 2, having found the required number to be 27, find $\frac{8}{9}$ of it. In this work the pupil is finding a fractional part of a number.

The work may also be proved by finding what fraction of the whole number the given part is.

In example 2, having found 27, find what fraction 24 is of 27. In this case, it is required to find what fractional part one number is of another.

In the following solve the example given, and in each case, by using the answer, make two other examples and solve.

6. 16 is $\frac{4}{5}$ of what number? *Ans.* 20.
What is $\frac{4}{5}$ of 20? *Ans.* 16.
What part of 20 is 16? *Ans.* $\frac{4}{5}$.

7. 24 is $\frac{3}{7}$ of what number?

8. $\frac{5}{9}$ of 36 are how many?

9. 15 is what part of 24?

10. What part of 18 is 8?

11. 22 is $\frac{2}{3}$ of what number?

12. What part is 8 of 20?

RAPID ORAL DRILL

13. $\frac{3}{8}$ of 24, 32, 48, 64, 56, 96, 72.

14. 24 is $\frac{3}{5}$ of —. $\frac{4}{5}$ of —. $\frac{6}{8}$ of —. $\frac{8}{9}$ of —.

15. 12 is what part of 16? 18? 20? 22? 32?

16. $\frac{2}{3}$ of $\frac{3}{4}$; of $\frac{6}{7}$; of $\frac{7}{8}$; of $\frac{8}{9}$; of $\frac{5}{12}$.

17. 8 is $\frac{3}{5}$ of —. $\frac{3}{4}$ of —. $\frac{3}{7}$ of —. $\frac{3}{9}$ of —.

18. $2\frac{1}{2}$ is $\frac{1}{2}$ of —. $\frac{1}{3}$ of —. $\frac{1}{4}$ of —? $\frac{1}{5}$ of —.

DECIMAL FRACTIONS, OR DECIMALS

140. **Study Recitation**

Fractions like $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{25}$, $\frac{8}{10}$, $\frac{25}{100}$, $\frac{125}{1000}$, in which both the numerator and denominator are written, are sometimes called **common fractions**.

Such common fractions as have 10, 100, 1000, or 1 with any number of 0's annexed, as a denominator, are also called **decimal fractions**.

Decimal fractions may be written by writing the numerator at the right of the decimal point (.) without writing the denominator, and when so written, may be called **decimals**.

In writing a decimal there must be as many figures at the right of the decimal point as there are 0's in the denominator, when it is written as a common fraction; if there is not this number of figures in the numerator, the required number is made up by placing 0's at the right of the decimal point before the numerator is written.

The decimal fractions below are first written as common fractions, to show the number of figures in the numerator and the number of 0's in the denominator. Below this, they are written as decimals, showing the same number of figures in the decimal as there are 0's in the denominator, and showing as many 0's at the right of the decimal point as the number of 0's in the denominator exceeds the number of figures in the numerator.

$$\begin{array}{ccccccc} \frac{1}{10} & \frac{7}{10} & \frac{25}{100} & \frac{5}{100} & \frac{1}{1000} & \frac{15}{1000} & \frac{235}{1000} \\ .1 & .7 & .25 & .05 & .001 & .015 & .235 \end{array}$$

$$\begin{array}{cccc} \frac{2}{10000} & \frac{47}{10000} & \frac{365}{10000} & \frac{1248}{10000} \\ .0002 & .0047 & .0365 & .1248 \end{array}$$

$$\begin{array}{ccc} \frac{5}{1000000} & \frac{325}{1000000} & \frac{7248}{1000000} \\ .000005 & .000325 & .007248 \end{array}$$

Learn the names of decimal places, or orders, at the right of the decimal point, as given below.

Tenths
Hundredths
Thousandths
Ten-thousandths
Hundred-thousandths
Millionths

.5
.25
.172
.2568
.14795
.234652

How many figures are required to write tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths?

Name the first order at the right of the decimal point; the 3d; the 6th; the 4th; the 2d; the 5th.

To the Teacher. In writing decimals, the chief difficulty occurs when pupils are required to write several numbers so that units of the same denomination shall be in the same column.

To avoid this difficulty, the pupil should know before beginning to write any decimal, how many figures are required to write the numerator as a whole number; how many places there are in the decimal; how many 0's there are between the decimal point and the numerator.

The order in writing should be: the decimal point first; the 0's, if any, next; the numerator last.

141. Drill

Whole Numbers. — How many figures are required to write units? Tens? Thousands? Hundred-thousands? Hundreds? Millions?

How many figures are required to write thirty-seven? Two hundred eight? Four hundred seventy-five? Ten thousand six hundred eighty-one? Five million seven hundred fifty-six thousand two hundred forty-six?

Decimals. — How many figures are required to write tenths? Thousandths? Millionths? Hundredths? Ten-thousandths? Hundred-thousandths?

How many figures are required to write eight tenths? Fifteen hundredths? Seven hundredths? One hundred thirty-five thousandths? Eight thousanths? Five hundred twenty-six ten-thousandths? Thirty-five hundred-thousandths? Two hundred fifty-seven millionths?

Read and add:

$$\begin{array}{l} .1 \\ .03 \\ .005 \\ .0006 \\ .00008 \\ \underline{.000004} \\ .135684 \end{array}$$

In the sum, beginning with the first order, give the value of the figure in each order, thus: 1 tenth, 3 hundredths, 5 thousandths, etc.

Read the entire sum.

WRITING AND READING DECIMALS

142. To write decimals:

Rule: *Write as many 0's at the right of the decimal point as the number of 0's in the denominator exceeds the number of figures in the numerator, and follow with the numerator.*

Study Recitation

1. Write twenty-five hundredths.

How many figures does it take to write twenty-five?

How many figures does it take to write hundredths?

As the number of figures is the same in both cases, write the decimal point with 25 at the right. Thus, .25.

2. Write two hundred fifty-six millionths.

How many figures are required to write two hundred fifty-six?

How many figures are required to write millionths?

How many ciphers must be used at the right of the decimal point?

143. To read decimals.

Rule: *Read the decimal as though it were a whole number and give it the name of the right-hand order.*

05 is read, *five*, because a 0 at the left of a whole number has no value; for the same reason 00015 is read, *fifteen;* 000241 is read two hundred forty-one.

In the decimal .05, the name of the right-hand order is hundredths; the decimal is read 5 hundredths.

In the decimal .00015, the name of the right-hand order is hundred-thousandths; the decimal is read 15 hundred-thousandths.

The decimal .000241 is read, 241 millionths.

Write the following decimals, one under the other, placing units of the same order in the same column. Add:

1. 5 tenths; 25 thousandths; 156 thousandths; 275 ten-thousandths.

2. 27 millionths; 350 hundred-thousandths; 27 thousandths; 4 hundredths.

3. One thousand hundred-thousandths; ten thousand six hundred one millionths; nine thousand eight hundred seventy-five hundred-thousandths; ninety-eight millionths; three hundred seven ten-thousandths; five tenths.

4. Two hundred ten-thousandths; nine hundred eighty-four millionths; nineteen thousandths.

5. 3074 millionths; 363 ten-thousandths; 125 thousandths; 24 hundredths; 5 tenths.

Read the following decimals:

6.	7.	8.	9.
.05	.27	.85664	.2706
.0005	.328	.472834	.34056
.00075	.04868	.275	.12545
.023	.2056	.0006	.980734
.07805	.20050	.00008	.0046
.27506	.206375	.000009	.7805

Add, and read the sums.

To the Teacher. For further drill in writing decimals, the teacher may dictate the above decimals and require pupils to write them.

A **mixed decimal** is a whole number and a decimal combined.

The mixed decimal 2.27 is read two and twenty-seven hundredths; the mixed decimal 24.056 is read twenty-four and fifty-six thousandths; the mixed decimal 5.8 is read five and eight tenths.

In reading mixed decimals, read the whole number first, add the word "and," then read the decimal, as: 256.008, read, two hundred fifty-six and eight thousandths.

Read the following mixed decimals:

10.	11.	12.	13.
2.028	27.027	375.0486	1256.078
3.004	39.0039	278.24	10450.0342
15.15	256.0256	506.0506	20005.00456

Add, and read the sums.

144. To change the denomination of a decimal:

$$.5 = \frac{5}{10} = \frac{1}{2}$$
$$.50 = \frac{50}{100} = \frac{1}{2}$$
$$.500 = \frac{500}{1000} = \frac{1}{2}$$
$$.5000 = \frac{5000}{10000} = \frac{1}{2}$$

What is the effect of annexing ciphers to a decimal?

Change:	Change to the same denomination:
1. .3 to thousandths	4. .027
2. .27 to thousandths	5. .55
3. .105 to ten-thousandths	6. .0048

7. How are decimals changed to the same denomination?

REDUCTION OF DECIMALS

145. To reduce a decimal to a common fraction in its lowest terms.

Study Recitation

1. Reduce .75 to a common fraction in its lowest terms.

$$.75 = \frac{75}{100} = \frac{3}{4}$$

Express the decimal as a decimal fraction, $\frac{75}{100}$. Then divide both numerator and denominator by 25. Or, divide both numerator and denominator by 5 and divide both terms of the resulting fraction by 5.

RULE: *Write the denominator of the decimal underneath the decimal. Remove the decimal point and reduce the fraction to its lowest terms.*

Reduce the following decimals to common fractions in lowest terms:

2. .5
3. .4
4. .25
5. .75
6. .30
7. .20
8. .10
9. .35
10. .40
11. .45
12. .50
13. .55
14. .375
15. .625
16. .875
17. .555
18. .125
19. .150
20. .225
21. .700
22. .025
23. .075
24. .235
25. .485
26. .3125
27. .5625
28. .0375
29. 4375
30. .0625
31. .8125
32. .0875
33. .6875
34. .0125
35. .1975
36. .1425
37. .3625

ADDITION AND SUBTRACTION OF DECIMALS

146. Rule: *Write the decimals so that units of the same order stand in the same column. Add or subtract as in whole numbers, placing the decimal point in the result under the decimal points above.*

Add: Read the numbers to be added and the sums:

1.	2.	3.
.275	.4	25.3
.056	.0045	1.75
.2346	.321	3.046
.105	.00065	.2507
.30852	.278	15.003

Subtract: Read minuend, subtrahend, and difference.

4.	5.	6.
2.0005	25.34	.9437
.2316	1.275	.72891

7.	8.	9.
35.04	1.027	5.
2.163	.9863	2.356

Review drill in writing decimals on pp. 230, 231.

10. Find the sum of twenty-five thousandths, three hundred four ten-thousandths, five hundredths, eighteen ten-thousandths.

11. Find the sum of 25 millionths, 305 ten-thousandths, 28 thousandths, 89 thousandths, 8 tenths.

12. Subtract four millionths from five thousandths.

13. Find the difference between 25 and 25 hundredths.

14. Subtract one thousandth from one tenth.

15. The sum of two numbers is 25 (.125); one of them is 7 (.018). What is the other?

Having the sum of two numbers and one of them how can you find the other?

16. Write for addition five decimals, each having a different number of decimal places. Read the decimals, add them, and read the sum.

17. $5.7 + 28.15 + 275.0463 + 198.25 - 21.32 = ?$

18. Give the rule for multiplication of decimals.

19. Give the rule for writing decimals.

20. Give the rule for reading decimals.

21. Give the rule for reducing a decimal to a common fraction in its lowest terms.

22. What is a decimal?

23. What is a fraction?

24. Beginning with 1 add 6's to 100.

25. Beginning with 2 add 7's to 100.

26. Beginning with 3 add 8's to 100.

27. Beginning with 4 add 9's to 100.

28 Beginning with 6 add 4's to 100.

To THE TEACHER. Continue the drill work in addition by substituting other numbers for 1, 2, 3, 4, and 6, in the above examples. Aim to develop accuracy and rapidity. Time pupils to see how quickly they can perform one or more of the examples given.

MULTIPLICATION OF DECIMALS

147. Rule: *Multiply as in whole numbers, and point off as many decimal places in the product as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

1. Multiply .025 by .05. Operation:

$$\begin{array}{r} .025 \\ \underline{.05} \\ .00125 \end{array}$$

Multiply:

2.	3.	4.	5.	6.	7.	8.
.275	1.05	27.05	.256	3.75	125	.256
.15	.08	.04	1.8	.004	.004	.001

9. What is the value of 8.5 tons of hay at $7.50 per ton?

10. At $.75 a day, how much will a boy earn in $4\frac{1}{2}$ days? Change $4\frac{1}{2}$ to a mixed decimal.

11. A field is 25.5 rods long, and 18.25 rods wide. How many square rods are there in the field? How many rods of fencing will it take to inclose the field? Work in two ways.

12. How many more rods of fencing will it take to divide the field into four oblong fields? Draw a figure showing how you have divided the field, and the additional fencing required.

13. A cubic foot of water weighs 62.5 pounds. How much do 9.25 cubic feet weigh?

148. To multiply a decimal by 10; 100; 1000, etc.

.25 × 10 = 2.50 = 2.5
.043 × 10 = 0.430 = .43
.5621 × 10 = 5.6210 = 5.621
.5 × 100 = 50.0 = 50.
2.75 × 1000 = 2750.00 = 2750.
4.8 × 100 = 480.0 = 480.

Notice that in every case the product can be obtained, without performing the multiplication, by removing the decimal point in the multiplicand as many places to the right as there are ciphers in the multiplier, annexing ciphers if necessary.

Multiply the following numbers by 10; by 100; by 1000.

1. 7.5 .25 .157 25.06
2. .0003 20. .046 2.03
3. 98.7 11.6 1.003 150.3
4. 1.003 25.6 40.009 10.05
5. .501 4.001 50.067 1.005
6. .004 13.75 3.02 .1005
7. Give the rule for multiplication of decimals.
8. Give the rule for writing decimals.
9. Give the rule for reading decimals.
10. Give the rule for reducing a decimal to a common fraction in its lowest terms.
11. What is a decimal?
12. What is a fraction?

DIVISION OF DECIMALS

149. When the divisor is a whole number.

Study Recitation

If 32 is divided by 4, the quotient is 8. If the quotient, 8, is multiplied by 4, the product, 32, is the same as the dividend.

The product of the divisor and quotient always gives the dividend; the dividend, therefore, has as many decimal places as there are decimal places in both the divisor and quotient. If the divisor is a whole number, the quotient has as many decimal places as the dividend.

ILLUSTRATIONS:

4)2.4	4)2.36	4).5268	12)27.984
.6	.59	.1317	2.332

In the above examples, the decimal point of the quotient is placed below the decimal point in the dividend before beginning the division. The first quotient figure is placed below the right-hand figure in the first partial dividend, and the other quotient figures below succeeding figures of the dividend.

4).24	4).0036	12).1104	12).0036
.06	.0009	.0092	.0003

In the above examples, the decimal point of the quotient is placed below the decimal point in the dividend before beginning the division. A 0 is placed below each figure of the dividend from the left until the last figure of the first partial dividend is reached; below this figure, the first significant figure of the quotient is placed, and the other figures in the quotient are placed in order below the remaining figures of the dividend.

Divide, placing the decimal point in the quotient before beginning the division, and prove the correctness of the work by multiplying the quotient by the divisor:

1. 8)3.84
2. 8)38.4
3. 9)2.457
4. 7)326.9
5. 12)26.04
6. 11)22.11
7. 6).276
8. 9).1152
9. 11).1001
10. 4).0028
11. 5).0275
12. 8).0224
13. 21)4.431
14. 32).1184
15. 42).2688
16. 18).0126
17. 16).0032
18. 14).0028

150. When the divisor is a decimal, or a whole number and a decimal.

Study Recitation

When the divisor is a decimal, or a whole number and a decimal, make it a whole number by moving the decimal point to the right of the right-hand figure, and move the decimal point in the dividend as many places to the right as it has been moved in the divisor.

.4)2.68 Make the divisor a whole number by moving the decimal point to the right of the 4, or one place to the right; move the decimal point in the dividend one place to the right, or to the right of 6; thus, .4.)2.6.8

When the decimal point is moved, extend the original decimal points downward in a short line, to show where they were in the example as given at first.

.24).048 Make the divisor a whole number; thus, ‸24 Move the decimal point in the dividend two places to the right; thus, ‸04.8

The example becomes ‸24.)‸04.8

.275)5.78 Make the divisor a whole number; thus, ‸275 Move the decimal point in the dividend three places to the right. As there are only two decimal places in the dividend, *annex* a 0, making three places; thus, 5‸780

The example becomes ‸275.)5‸780

0's may be annexed to a decimal without changing its value. In moving the decimal point to the right in the dividend, if there are not enough places, annex the 0's to make required number of decimal places. When the last figure of the dividend has been used, if the division is not exact, the remainder may be written as a remainder is written in division of whole numbers, or 0's may be annexed and the division may be continued as far as may be desired.

In the following exercise, first change all the divisors to whole numbers, and make corresponding changes in the dividends; then divide.

WRITTEN

1. .27)754.65 changed as directed becomes ‸27.)754‸65

2. 11.5)46 3. 24.75)99 4. 4.125)35.0625 5. 7.5)18

Perform the indicated division:

6. .66 ÷ .015 8. 5.2 ÷ .005 10. 12.75 ÷ 2.5

7. 1.69 ÷ .0013 9. .84 ÷ 1.2 11. 3.91 ÷ 1.7

12. 17.5)40.25
13. 4.2)4704
14. 4.425).2832
15. .0115)80.5
16. .84)9.408
17. .032)25.6
18. .125)8.75
19. 8.44).633
20. 8.75)3.85

PROBLEMS

151. WRITTEN

1. A boat goes at the rate of 17 (7.25) miles per hour. In how many hours will it go 85 (65.25) miles?

2. At $.37 per bushel, how many bushels of potatoes can be bought for $4.81?

In multiplying or dividing a number expressing dollars and cents, treat the number as a mixed decimal.

3. Into how many building lots, each containing .125 of an acre, can 9 acres be divided?

4. If the average rainfall is 2.72 inches per month, in how many months does the rainfall amount to 32.64 inches?

5. If a boy earns $12.25 a week, in how many weeks will he earn $122.50?

6. If 9 tons of soft coal cost $ $32.62\frac{1}{2}$, how much will 1 ton cost?

7. Write 15¢ (3¢) as a decimal of a dollar. Write 62¢ as a decimal of a dollar.

8. How many yards of carpeting at $ 1.75 a yard, can be bought for $ 87.50?

9. If a train travels .6 of a mile a minute, how many minutes will it take to travel 20.4 miles?

152. To divide a decimal or a whole number by 10, 100, or 1000, or 1 with any number of ciphers annexed.

1. How is a decimal multiplied by 10, 100, 1000, etc.? See p. 237.

2. Multiply the following decimals by 10: 2.5; .27; 3.105. If each of the products is divided by 10, what are the quotients? What is the shortest way of dividing by 10?

3. Multiply the following numbers by 100: 3.56; 2.7; 1.03. If each of the products is divided by 100, what are the quotients? How is a decimal divided by 100? By 1000?

4. How do we multiply a decimal by 1 with any number of ciphers annexed?

5. How do we divide a decimal by 1 with any number of ciphers annexed?

Any whole number may be written with a decimal point at the right of units; thus, 25.; 375.; 123.

Whole numbers may be divided by 10, 100, 1000, etc., in the same way as decimals.

Divide by 10; by 100; by 1000:

6.	2.78	50.4	5.675	25	5275
7.	10.07	2.9	27.80	275	470.5
8.	253	275	27.15	126	126.5
9.	25.3	27.5	271.5	22.6	12.65
10.	2.53	2.75	2.715	1.26	1.265

153. To reduce a common fraction to a decimal.

RULE: *Divide the numerator by the denominator.*

Study Recitation

1. Reduce $\frac{3}{4}$ to a decimal.

$$\frac{3}{4} = 3 \div 4 = 4\overline{)3.00} \atop \underline{\quad} .75$$

What must be done to 3 before it can be divided by 4?

After annexing two 0's, divide as in § 150. The answer is .75.

2. Reduce $\frac{4}{7}$ to a four-place decimal.

$$7\overline{)4.0000} \atop .5714\frac{2}{7}$$

Since $\frac{4}{7} = 4 \div 7$ we annex four 0's and divide as indicated.

Reduce to decimals, not beyond four places:

3. $\frac{1}{4}$
4. $\frac{1}{5}$
5. $\frac{1}{3}$
6. $\frac{1}{8}$
7. $\frac{3}{4}$
8. $\frac{5}{8}$
9. $\frac{4}{5}$
10. $\frac{7}{20}$
11. $\frac{8}{15}$
12. $\frac{7}{8}$
13. $\frac{5}{6}$
14. $\frac{5}{16}$
15. $\frac{3}{8}$
16. $\frac{4}{15}$
17. $\frac{13}{20}$

BILLS

154. When one person buys goods from another, the seller makes out a statement of the amount and value of the goods of each kind, the total value of the goods, and the date of the sale, and gives this statement to the buyer. Such a statement is called a **bill** or **invoice.**

A **debt** is something one owes to another.

The **debtor** is the person who owes the debt.

The **creditor** is the person to whom the debt is owed.

The **buyer** is debtor to the **seller** for the value of the goods bought. The **seller** is the buyer's **creditor** for the value of the goods sold.

1. Mrs. John Smith bought of Carter & Co., of Chicago, Apr. 10, 1908, 8 yd. of silk @ $2.48; 9 collars @ 25¢; 6 pr. of shoes @ $3.25; 3 pr. of gloves @ $1.75, and received the following bill:

Chicago, Ill., Apr. 10, 1908.

Mrs. John Smith

Bought of Carter & Co.

Apr.	10	8 yd. Silk	@ $2.48	19	84
		9 Collars	@ .25	2	25
		6 pairs Shoes	@ 3.25	19	50
		3 pairs Gloves	@ 1.75	5	25
				46	84

Received Payment,
Carter & Co.

The words "Received Payment," with the name of the firm selling the goods, show that the goods were paid for.

The above is a receipted bill.

2. James Johnson,
Bought of Wm. Smith & Sons.
5 lb. Coffee @ 30¢ ____________
8 lb. Dried Beef @ 16¢ ____________
12 lb. Sugar @ 6¢ ____________
2 lb. Starch @ 12¢ ____________

Received Payment,
Wm. Smith & Sons.

Find the cost of each item, and fill it in at the right. What is the amount of the bill?

3. Suppose James Johnson on the same day bought of Anderson & Co., dry goods merchants, the following articles: 12 yd. cotton cloth @ 7¢ per yard; $5\frac{1}{2}$ yd. silk @ $1.20 per yard; $8\frac{3}{4}$ yd. ribbon @ 12¢ per yard.

Make out and receipt this bill, following form 1.

Make out and receipt bills for the following:

4. A. L. Wilson bought of Kruse & Co., Kansas City, Mo., Jan. 5, 1909, 25 lb. of bacon @ 15¢; 25 lb. of lard @ 13¢; $9\frac{1}{2}$ lb. of sausage @ 18¢.

In the above bill, which party owes a debt? To which party is the debt owed? Who is the debtor? Who is the creditor?

5. John Smith, a gardener, works 3 days for Henry Jones @ $1.50 per day; he furnishes 10 packages of garden seed @ 10¢ each; a hoe @ 75¢, and pays a man $1.25 for plowing the garden.

6. You buy from James Peterson on the day your school opens this year, a geography for $1.25; an arithmetic for 60¢; a speller for 25¢; 3 lead pencils for 10¢; 2 pads of paper @ 5¢ each.

Give results orally:

7. $\frac{3}{8}+\frac{1}{4}=?$ $\frac{5}{7}+\frac{1}{2}=?$ $\frac{3}{5}+\frac{2}{3}=?$ $\frac{5}{6}+\frac{1}{2}=?$

8. $\frac{3}{4}-\frac{1}{8}=?$ $\frac{4}{9}-\frac{1}{3}=?$ $\frac{5}{12}-\frac{3}{8}=?$ $\frac{7}{16}-\frac{1}{8}=?$

9. $\frac{4}{5}\times\frac{3}{8}=?$ $\frac{12}{18}\times\frac{9}{4}=?$ $\frac{6}{7}\times\frac{2}{3}=?$ $\frac{5}{14}\times\frac{7}{9}=?$

10. $\frac{8}{9}\div\frac{2}{3}=?$ $\frac{12}{15}\div\frac{3}{5}=?$ $\frac{7}{20}\div\frac{4}{5}=?$ $\frac{9}{24}\div\frac{5}{8}=?$

155. DENOMINATE NUMBERS

An **abstract number** is a number used without reference to any particular thing; as, 4, 6, 8, 20.

A **concrete number** is a number used with the name of some thing; as, 4 bushels, 6 horses, 8 apples, 20 pounds.

A **denominate number** is a concrete number used with the name of some unit of measurement; as, 4 ounces, 6 hours, 8 feet, 20 pounds.

156. DRY MEASURE

Dry measure is used in measuring different kinds of vegetables, fruits, and other dry commodities.

TABLE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

ORAL

1. How many pints equal 2 qt.? 4 qt.? 9 qt.? 7 qt.?

2. How many quarts equal 8 pt.? 12 pt.? 24 pt.? 14 pt.?

3. How many quarts equal 4 pk.? 6 pk.? 9 pk.? 12 pk.?

4. How many pecks equal 16 qt.? 24 qt.? 48 qt.? 32 qt.?

5. How many pecks equal 8 bu.? 2 bu.? 12 bu.? 9 bu.?

6. How many bushels equal 16 pk.? 32 pk.? 12 pk.?

7. How many quarts $= \frac{1}{2}$ pk.? How many pints?

8. How many pecks = 3 bu.? 3 bu. 2 pk.?

9. How many bushels and pecks = 9 pk.? 13 pk.?

10. How many quarts = 5 pk.? 5 pk. 5 qt.? 7 pk.?

11. How many pecks and quarts = 17 qt.? 23 qt.?

12. How many quarts = 4 pk.?

Analysis.—1 pk. = 8 qt. 4 pk. = 4 times 8 qt. = 32 qt.

13. How many bushels = 17 pk.?

Analysis.—4 pk. = 1 bu. 17 pk. = as many bushels as 4 is contained times in 17, or $4\frac{1}{4}$ bu. = 4 bu. 1 pk.

14. How many quarts = 3 pk. 5 qt.?

Analysis.—1 pk. = 8 qt.; 3 pk. = 3 times 8 qt., or 24 qt. 24 qt. + 5 qt. = 29 qt.

Require analysis of examples 1–11 to secure accuracy and readiness of statement of operations.

15. How do we reduce bushels to pecks? Pecks to bushels? Pecks to quarts? Quarts to pecks? Quarts to pints? Pints to quarts?

It is not a sufficient answer to these questions to say "multiply" or "divide."

Written

In the table of Dry Measure, what is the lowest denomination? The highest? By what operation is a number of one denomination changed to a lower denomination? To a higher denomination?

In the reduction of denominate numbers, decide in order, whether the reduction is to a lower or to a higher denomination; whether the operation is multiplication or division; by what number to multiply or divide. Perform the required operation.

Reduce:

16. 5 bu. to pecks; 25 bu. to pecks; 198 bu. to pecks.

17. 3 qt. to pints; 38 qt. to pints; 59 qt. to pints.

18. 4 bu. 3 pk. to pecks; 24 bu. 3 pk. to pecks.

19. 49 qt. to pecks; 95 pt. to quarts; 287 qt. to pecks.

20. Reduce 8 bu. to pecks, the pecks to quarts, and the quarts to pints.

Give results orally and rapidly:

21. 5×16, 7×14, 8×23, 9×47, 8×24.

22. 7×32, 5×64, 7×56, 8×29, 5×63.

23. $68 \div 8$, $127 \div 9$, $256 \div 8$, $324 \div 4$, $250 \div 5$.

24. $79 \div 6$, $56 \div 4$, $144 \div 8$, $272 \div 9$, $345 \div 9$.

157. LONG MEASURE

Long measure is used in measuring length or distance. Long measure is sometimes called linear measure.

TABLE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ ft. = 1 rod (rd.)
$\left.\begin{array}{l}\text{320 rd.}\\ \text{1760 yd.}\\ \text{5280 ft.}\end{array}\right\}$ = 1 mile (mi.)

Oral

1. How many inches = 2 ft.? 5 ft.? 3 ft.? 7 ft.?
2. How many feet = 3 yd.? 7 yd.? 2 yd.? 12 yd.?
3. How many feet = 36 in.? 60 in.? 96 in.?
4. How many yards = 12 ft.? 18 ft.? 27 ft.?
5. How many inches = 1 yd.? 6 ft.? 2 yd.?
6. How many inches = 1 ft. 8 in.? 2 ft. 3 in.?
7. How many feet = 2 yd.? 2 yd. 2 ft.? 3 yd.?
8. How many feet and inches = 26 in.? 39 in.?
9. How many yards and feet = 17 ft.? 26 ft.?
10. How many rods = $\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.? $\frac{1}{8}$ mi.?
11. How many feet = 2 rd.? 2 rd. 10 ft.?
12. What part of 1 ft. are 6 in.? 8 in.? 4 in.?
13. What part of 1 yd. is 1 ft.? What part of 1 yd. are 2 ft.? $1\frac{1}{2}$ ft.?
14. What part of 36 in. are 18 in.? 20 in.? 30 in.?
15. What part of 1 yd. are 1 ft. 6 in.? 2 ft. 6 in.?

Written

16. How many yards = 27 ft.? 270 ft.? 455 ft.?
17. How many yards and feet = 32 ft.? 325 ft.?
18. How many feet and inches = 38 in.? 386 in.?
19. How many feet = 5 yd. 2 ft.? 25 yd. 2 ft.?

20. How many feet = 2 rd.? 7 rd.? 9 rd.?

21. How many yards = 3 rd.? 5 rd.? 8 rd.?

22. How many rods = $16\frac{1}{2}$ yd.?

OPERATION: $16\frac{1}{2} \div 5\frac{1}{2} = 3$.

23. How many rods = 18 yd.? $27\frac{1}{2}$ yd.? 42 yd.?

24. How many rods = 33 ft.?

WORK: $33 \div 16\frac{1}{2} = 2$.

25. How many rods = 128 ft.? $148\frac{1}{2}$ ft.? 198 ft.?

26. How many rods = 2 mi.? 3 mi. 120 rd.?

27. How many rods = $3\frac{1}{2}$ mi.? $2\frac{1}{4}$ mi.? $5\frac{3}{8}$ mi.?

28. Find the sum of the numbers from 1 to 13 inclusive.

29. Subtract 397 from 2464, successively, until the remainder is less than 397. What is the remainder?

30. Reduce a fraction to its lowest terms, and give the rule.

31. Reduce a mixed number to an improper fraction, and give the rule.

32. Reduce an improper fraction to a mixed number, and give the rule.

33. Change a fraction to another fraction of a higher denomination, and give the rule.

34. What is a mixed number? An improper fraction? When is a fraction in its lowest terms?

What is a multiple? A common multiple? Least common multiple? What is a prime number? Give the prime numbers below 20.

LIQUID MEASURE

158. Liquid measure is used in measuring liquids.

Table

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)

$$\left.\begin{array}{r}4\text{ quarts}\\ 8\text{ pints}\\ 32\text{ gills}\end{array}\right\} = 1\text{ gallon (gal.)}$$

Note. The pint and quart measures used in measuring liquids are a little smaller than the pint and quart measures in Dry Measure. The actual measures should be compared.

Oral

1. How many gills = 2 pt.? 4 pt.? 9 pt.?
2. Reduce to pints: 20 gi.; 36 gi.; 48 gi.
3. Reduce to gills: 5 pt. 2 gi.; 8 pt. 3 gi.
4. How many quarts = 8 pt.? 16 pt.? 24 pt.?
5. How many quarts = 7 gal.? 9 gal.? $12\frac{1}{2}$ gal.?
6. Reduce to pints: 7 qt.; 7 qt. 1 pt.; 5 qt. 1 pt.
7. Reduce to pints and quarts: 9 pt.; 19 pt.
8. Reduce to quarts: 4 gal.; 4 gal. 2 qt.; 3 gal.
9. Reduce to quarts and gallons: 27 qt.; 13 qt.

10. What part of 1 pt. is 1 gi.? Are 3 gi.? 2 gi.?

11. 1 qt. is how many pints? What part of a quart are 3 gi.? 5 gi.?

12. What part of 5 qt. are 5 pt.? 10 pt.? 8 pt.?

13. How many pints = 1 gal.? What part of a gallon are 3 pt.?

WRITTEN

14. Reduce 27 gal. 3 qt. to quarts.

$$\begin{array}{ll}
\text{gal.} & \text{qt.} \\
27 + & 3 \\
\underline{4} & \\
108, & \text{number of qt.} \\
\underline{3} & \text{qt.} \\
111 & \text{qt.}
\end{array}$$

ANALYSIS: There are 4 qt. in 1 gal.; in 27 gal. there are 108 qt.; 108 qt. + 3 qt. = 111 qt.

15. Reduce 57 qt. 1 pt. to pints.

16. Reduce 257 qt. to gallons.

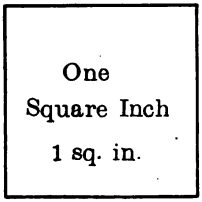
17. How do we reduce gallons to quarts? Quarts to pints? Pints to quarts? Quarts to gallons? Gills to pints? Pints to gills?

18. By what operation is a denominate number reduced to a lower denomination? To a higher denomination?

19. Give the table of Dry Measure; of Long Measure; of Liquid Measure.

SQUARE MEASURE

159. Square measure is used in measuring surfaces. A square is a surface bounded by four equal lines, and having four square corners. The lines at the corners make right angles. The figure at the right has four equal sides and four right angles or square corners. It is a square. As each side is 1 in. long, the surface inside the lines is a square inch.



A square inch is a square surface 1 in. long and 1 in. wide.

A square foot is a square surface 1 ft. long and 1 ft. wide.

A square yard is a square surface 1 yd. long and 1 yd. wide.

Draw each of these squares on the board.

Divide the square foot into square inches.

Divide the square yard into square feet.

A square rod is a square surface 1 rd. long and 1 rd. wide. With a tape measure or a line 1 rd. long, lay out a square rod on the floor of the schoolroom and one on the school grounds.

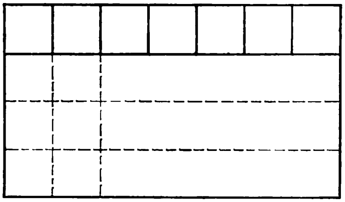
TABLE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod or perch (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Oral

1. How many square feet = 5 sq. yd.? 6 sq. yd.?
2. How many square yards = 27 sq. ft.? 81 sq. ft.?
3. How many square inches = $\frac{1}{2}$ sq. ft.? $\frac{1}{4}$ sq. ft.?
4. 12 sq. in. are what part of a square foot?
5. 24 sq. in. are what part of a square foot?
6. 3 sq. ft. are what part of a square yard?
7. 7 sq. ft. are what part of a square yard?
8. How many square rods equal $\frac{1}{2}$ A.? $\frac{1}{8}$ A.? $\frac{3}{8}$ A.?

Study Recitation



If this figure is 7 in. long and 4 in. wide, how many square inches are there in the upper row of squares? How many inch rows are there? How many square inches are there in its surface?

If there are 28 sq. in. in the surface of the rectangle, and it is 4 in. wide, how long is it? If a strip 1 in. long is marked off across one end of the rectangle, how many square inches are there in its surface? For each inch of length of the rectangle, how many square inches are there?

How many square inches are there in the area of the rectangle? In one inch of its length how many square inches are there? Knowing the area of the rectangle in square inches, and the number of square inches in 1 in. of the length of the rectangle, how can you find the length?

Written

9. How many square inches = 7 sq. ft.? 9 sq. ft.?

10. How many square feet = 45 sq. yd.? 7 sq. yd.?

11. How many square yards and square feet = 65 sq. ft.? 278 sq. ft.? 356 sq. ft.? 215 sq. ft.?

12. How many square inches = 3 sq. ft., 27 sq. in.?

13. A floor is 16 ft. wide and 20 ft. long. How many square feet are there in its surface? How many square yards?

14. A garden is 6 rd. long and 4 rd. wide. How many square rods are there in its area? How many square feet in its area?

15. If the garden is laid out in beds 1 rd. long and 11 ft. wide, how many such beds are there in the garden? Show by diagram.

CUBIC OR SOLID MEASURE

Study Recitation

160. Cubic measure is used in measuring solids.

A solid with square corners, and 1 inch long, 1 inch wide, and 1 inch high is a **cubic inch.**

Show that on a square foot along one side, 12 cubic inch blocks can be placed in a row.

What are the dimensions of the solid formed by the 12 cubic inch blocks?

Show the number of rows of 12 blocks laid side by side, required to cover the square foot surface.

What are the dimensions of the solid thus formed? How many cubic inch blocks are there in this solid?

If another layer of blocks in rows is placed so as to cover the first layer entirely, what will be the dimensions of the solid thus formed?

How many cubic inch blocks will this solid contain?

If a third layer is placed upon the other two layers, what will be the height of the solid thus formed? What its length and breadth? How many cubic inch blocks does it contain? How many blocks are there in each layer? How many such layers must there be to make the solid 1 ft. high? How many cubic inch blocks are there in the solid 1 ft. long, 1 ft. wide, and 1 ft. high?

A solid 1 foot long, 1 ft. wide, and 1 ft. high is a **cubic foot.**

Show the number of cubic foot blocks that must be placed upon a square yard surface to make a solid 3 ft. long, 3 ft. wide, and 1 ft. high; 2 ft. high; 3 ft., or 1 yd. high. A solid 1 yd. long, 1 yd. wide, and 1 yd. high is a **cubic yard.**

TABLE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high is 1 cd.

Oral and Written

1. How many cubic inches are there in a solid 3 ft. long, 1 ft. wide, and 1 ft. high?

2. How many cubic feet equal 18 cu. yd.?

3. A pile of 2 ft. wood is 8 ft. long and 4 ft. high. What part of a cord is it? Solve in two ways.

4. A pile of 16 in. wood is 8 ft. long and 4 ft. high. What part of a cord is it?

5. How many cubic feet are there in a solid 4 ft. long, 3 ft. wide, and 1 ft. high?

How do you find the cubic feet in problem 5?

6. If a solid is 4 ft. long, 3 ft. wide, and 2 ft. high, how many cubic feet does it contain? How do you find the result?

7. How many cubic feet are there in $\frac{1}{2}$ cd.? In $\frac{3}{8}$ cd.? In $\frac{7}{8}$ cd.?

8. How many cubic feet are there in a room 12 ft. long, 10 ft. wide, and 9 ft. high?

9. How many square feet are there in the floor? In each end? In each side?

10. How many square feet of linoleum will it take to cover a floor 12 ft. long by 9 ft. wide? How many square yards?

REVIEW

161. 1. When the length and breadth of a rectangular surface are given in feet, how do you find the area? Give an illustration.

2. If the length and breadth of a rectangular field are given in rods, how do you find the area? Illustrate by an example. What is the denomination of the answer?

3. When the number of square rods in a field is known, how do you find the number of acres in the field? Give an example to illustrate.

4. When the number of square feet in the area of any surface is given, how do you find its area in square yards? Give an example to illustrate.

5. When the length and width of a rectangle are given in feet, how do you find the number of square yards it contains? Illustrate.

6. Having the length of a field in rods, how do you find its length in feet? Illustrate.

7. A room is 5 yd. long and 4 yd. wide. How many square yards of carpeting will it take to cover the floor?

8. A man has a pile of 4-ft. wood, 36 ft. long and 4 ft. high. If he pays $.75 a cord for cutting it, how much does he pay for cutting the entire pile? If he sells the wood for $7.50 per cord, how much does he

receive for it? How much has he left after deducting the cost for cutting?

9. How many rods = 1 mi.? How do you find the number of feet in this number of rods?

10. A man walks $2\frac{1}{2}$ mi. an hour. How many rods does he walk in 3 hr.?

11. A pail holds 3 gal. 2 qt. of water. How many times can a quart measure be filled from the pail? A 2-qt. measure? How many quarts of water will it take to fill the pail 4 times? How many gallons?

12. If cream sells at 10¢ a pint, how much will a milkman receive per quart? Per gallon?

13. If milk sells for 6¢ a quart, what is the value of a 10-gal. can of milk?

14. A man walks $3\frac{1}{2}$ mi. per hour for 3 hr. How many rods does he travel?

15. How do you reduce gallons to quarts? Give an example and analyze it.

16. How do you reduce inches to feet? Feet to yards? Inches to yards?

17. How do you reduce yards to feet? Feet to rods? Miles to rods?

18. 7 sq. ft. is what part of 2 sq. yd.?

19. A surface 2 ft. by 4 ft. is what part of a surface 4 ft. square? Work in two ways.

20. A surface 3 ft. by 5 ft. is what part of a surface 6 ft. by 10 ft.? Show by a diagram and work in two ways.

21. If the length, width, and thickness of a solid are given in feet, how do you find the number of cubic feet it contains? Having found the number of cubic feet it contains, how do you find the number of cubic yards it contains?

22. Name the denominations of Long Measure; of Square Measure; of Cubic Measure.

AVOIRDUPOIS WEIGHT

162. Avoirdupois weight is used in weighing most articles bought and sold by weight.

TABLE

16 ounces = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight } = 1 ton (T.)
2000 pounds }

Many scales do not show weight less than 1 oz. Some show divisions of the ounce, as $\frac{1}{2}$ or $\frac{1}{4}$ oz.

ORAL AND WRITTEN

1. How many ounces = 2 lb.? 4 lb.? 25 lb.? $18\frac{1}{2}$ lb.?

2. How is a pound reduced to ounces?

3. A butcher sells 3 lb. 4 oz. of steak at 20¢ per pound. How much does he receive for it?

4. How many cwt. = 3000 lb.? 2575 lb.?

5. How do you reduce pounds to hundredweight? How do you divide by 100?

6. What part of a ton are 100 lb.? 400 lb.? 500 lb.?

7. How much will 4500 lb. of hay cost at $8.00 per ton?

8. How much will 1000 lb. of coal cost at $7.50 per ton?

9. A man distributes 15 cwt. of flour among some poor families; to the first family he gives $\frac{1}{10}$ of it, to the next $\frac{1}{15}$ of it, and to the next 3 families together $\frac{1}{5}$ of it. The remainder he divides equally among 5 families. How many pounds do each of these 5 families receive? Make other questions on the problem.

MISCELLANEOUS TABLES

163.

Counting Table

20 things = 1 score
12 things = 1 dozen (doz.)
12 dozen = 1 gross

Paper Table

24 sheets of paper = 1 quire (qr.)
20 quires = 1 ream

Oral

1. A man's age is threescore years and ten. How old is he?

2. How much are $6\frac{1}{2}$ doz. eggs worth at 12¢ a dozen?

3. A dealer buys 2 gross of steel pens. At 10¢ per dozen, how much do they cost? If he sells the pens at 1¢ each, how much does he gain on the lot?

4. If clothespins are bought at 4¢ per dozen, how much will $5\frac{3}{4}$ gross cost?

5. A grocer sells 3350 oranges at 30¢ a dozen. How much does he get for the lot?

6. A girl uses 1 qr. of paper every 2 wk. How long will it take her to use 1 ream?

7. If a boy uses 2 qr. of paper every 2 wk., how long will it take him to use 2 reams?

TIME MEASURE

164.

TABLE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

The common year has 365 da. Leap year has 366 da.

Learn: "Thirty days have September,
April, June, and November,
All the rest have thirty-one,
Save February, which alone has twenty-eight,
And one day more we add to it one year in four."

The year 1908 is a leap year. February has 29 days in 1908, 1912, 1916, etc.

Oral

1. How many minutes = 2 hr.? $2\frac{1}{2}$ hr.? $4\frac{1}{3}$ hr.?

2. How many hours are there from midnight to noon? From noon to midnight? From 8 o'clock to 12? From 12 to 4? From 8 to 4? From 6 A.M. to 2 P.M.?

3. How many weeks and days are there in a common year? In a leap year?

4. How many days are there in the spring months? In the summer months?

5. What part of 90 min. are 60 min.? What part of an hour are 25 min.?

6. What part of 2 hr. are 100 min.? What part of 2 hr. are $1\frac{1}{2}$ hr.?

7. 1 wk. is what part of the month of June? Of July? Of Feb., 1908? Of Feb., 1909?

REDUCTION OF SIMPLE DENOMINATE NUMBERS

165. Reduction of simple denominate numbers is the process of changing a number of one denomination to units of another denomination.

Oral and Written

1. How are pints reduced to quarts? By dividing the given number of pints by the number of pints in a quart.

2. How are quarts reduced to pecks?

3. How are pecks reduced to bushels?

4. In 72 in. how many feet are there? In 6 ft. how many yards are there?

5. How are inches reduced to feet?

6. How are feet reduced to yards?

7. How are rods reduced to miles?

8. In 288 sq. in. how many square feet are there?

9. In 36 sq. ft. how many square yards are there?

10. In 320 sq. rd. how many acres are there?

11. How are square inches reduced to square feet?

12. How are square feet reduced to square yards?

13. How are square rods reduced to acres?

In each of the foregoing cases, the change has been from a given denomination to a higher denomination.

14. Give and solve examples in reducing from a higher to a lower denomination in Dry Measure.

15. In Long Measure. **17.** In Liquid Measure.

16. In Square Measure. **18.** In Time Measure.

19. Reduce 216 in. to yards.

20. Reduce 11 yd. to rods.

21. Reduce 960 rd. to miles.

22. Reduce 6000 lb. to tons.

23. Reduce 5 yd. to feet; 60 ft. to inches.

24. Reduce 5 yd. to inches; 8 yd. to inches; 2 rd. to feet; 33 ft. to inches.

25. Reduce 2 sq. yd. to square feet; 18 sq. ft. to square inches. Reduce 2 square yd. to square inches.

166. To reduce a denominate number to the next lower denomination.

RULE: *Multiply the given number by the number of units of the next lower denomination it takes to make* 1 *unit of the given denomination.*

167. To reduce a denominate number to the next higher denomination.

RULE: *Divide the given number by the number of units of the given denomination it takes to make* 1 *unit of the next higher denomination.*

REDUCTION OF COMPOUND DENOMINATE NUMBERS

168. **Reduction of compound denominate numbers** is a change from units of two or more denominations to units of one denomination, or a change from units of one denomination to units of two or more denominations.

1. Reduce 7 bu. 3 pk. to pecks.

2. Reduce 5 pk. 4 qt. to quarts.

3. In 4 rd. 8 ft. how many feet are there?

4. In 44 rd. 13 ft. how many feet are there?

5. A man divides a piece of ground containing 2 A. into lots each containing 32 sq. rd. How many lots are there in the piece? If each of the lots is 8 rd. long, how wide are they?

6. A room is 3 yd. 2 ft. wide and 5 yd. 1 ft. long; what are its dimensions in feet? Draw the floor of the room on a scale of 1 in. to the foot. How many square feet are there in the floor? How many square yards? If the room is 10 ft. high, what are the dimensions of each end? Of each side? What is the area of each end? Of each side? Of the four sides?

7. A piece of cloth 1 yd. wide is 3 ft. long. What is the area in square feet?

8. From cloth 1 yd. wide, a piece 2 ft. long is cut off; what part of a square yard is cut off? Show by a figure drawn on the board. How many square feet are in the piece cut off? How many square feet are there in 1 sq. yd.? What part of a square yard is the piece cut off?

9. A piece of cloth is 12 yd. long; if a piece 6 yd. 2 ft. long is cut off, what part of the strip is the piece cut off? What part of the strip is the remaining part? Make and solve a similar example.

10. If you have the length of a room in feet and its width in inches, how shall you find its area in square inches?

To reduce to higher denominations.

169. ORAL AND WRITTEN

1. In 24 in. (276 in.) how many feet are there?

2. In 256 in. how many feet and how many inches over are there?

3. In 29 ft. how many yards and how many feet over are there?

4. Reduce 38 ft. to yards and feet.

5. Reduce 35 qt. to pecks and quarts.

6. Reduce 29 pk. to bushels and pecks.

7. Reduce 18 sq. ft. to square yards.

8. Reduce 38 sq. ft. to square yards and square feet.

9. Reduce 47 qt. to gallons and quarts.

10. Reduce 90 min. to hours and minutes.

11. Reduce 48 hr. to days.

12. Reduce 50 hr. to days and hours.

13. Reduce 14 da. to weeks.

14. Reduce 36 da. to weeks.

15. In 24 oz. how many pounds and ounces are there?

16. A man sells 60 eggs at 10¢ a dozen. How much does he receive for them?

17. If he had sold 54 eggs at the same rate, how much would he have received?

REVIEW

170. Answer, and solve an original example to illustrate.

1. How are quarts reduced to pecks?

2. How are feet reduced to inches?

3. How are months reduced to years?

4. How are minutes reduced to hours?

5. How are hours reduced to days?

6. How are days reduced to hours?

7. How are hours reduced to minutes?

8. How are pounds reduced to ounces?

9. How are gallons reduced to quarts?

10. How are pints reduced to quarts?

11. Give the rule for reducing a number of any denomination to a higher denomination.

12. Give the rule for reducing a number of any denomination to a lower denomination.

ADDITION OF COMPOUND DENOMINATE NUMBERS

171. WRITTEN

1. Add 3 pk. and 2 pk.

The sum is 5 pk. 5 pk. = 1 bu. 1 pk.

2. Add 7 pk. and 3 pk.

The sum is 10 pk. 10 pk. = 2 bu. 2 pk.

	3.	4.	5.	6.	7.
Add:	5 pk.	7 qt.	3 qt.	5 pk.	9 oz.
	2 pk.	6 qt.	7 qt.	6 pk.	6 oz.
	1 bu. 3 pk.	1 pk. 5 qt.			

8.

$$\begin{array}{r} \text{bu. pk.} \\ 4+5 \\ \underline{9+6} \\ 15+3 \end{array}$$

6 pk. and 5 pk. are 11 pk. = 2 bu. 3 pk. Write the 3 pk. under the pk. column and add the 2 bu. to the next column.

9.	**10.**	**11.**	**12.**
bu. pk.	bu. pk.	bu. pk.	bu. pk
12 + 4	12 + 5	215 + 3	26 + 7
5 + 6	18 + 4	156 + 7	15 + 7

13.	**14.**	**15.**	**16.**
pk. qt.	pk. qt.	pk. qt.	pk. qt.
5 + 6	1 + 3	2 + 6	2 + 5
7 + 5	3 + 6	3 + 5	9 + 7

17.	**18.**	**19.**	**20.**
bu. pk.	yd. ft.	rd. yd.	lb. oz.
8 + 3	5 + 2	8 + 4	5 + 8
2 + 2	3 + 1	7 + 5	6 + 9

21. A grocer sells one customer 5 lb. 9 oz. of butter; another customer, 8 lb. 10 oz. How much does he sell to both?

22. The running time of a train between stations A and B is 2 hr. 35 min.; between stations B and C, is 1 hr. 40 min. If the train stops 3 min. at station B, what is the time from A to C? If the train leaves A at 7.15 A.M., at what time does it arrive at C? at B? At what time does it leave B?

23. A room is 12 ft. 6 in. wide and 18 ft. 9 in. long. What is the distance around it?

24. From a bolt of ribbon a merchant sells to one person $3\frac{1}{2}$ yd., to another $2\frac{1}{4}$ yd. How many yards, feet, and inches does he sell?

25. In one pile of wood there are 3 cd. 80 cu. ft.; in another pile, 15 cd. 112 cu. ft. How much wood is there in both piles? Give three answers: 1st, cords and cubic feet; 2d, cords and fraction of a cord; 3d, cords and decimal of a cord.

26. One field contains 10 A. 40 sq. rd., another 25 A. 140 sq. rd. How much land is there in both fields? Work first, by adding the denominate numbers; then by first reducing the denominate numbers to mixed numbers.

172. To add compound denominate numbers as mixed numbers.

Reduce the following to mixed numbers, and add the mixed numbers of the same denomination:

1.	2.	3.	4.	5.	6.
bu. pk. bu.	bu. pk.	qt. pt.	gal. qt.	ft. in.	yd. ft.
$8+3=8\frac{3}{4}$	$7+1$	$8+1$	$4+3$	$4+6$	$5+2$
$5+2=5\frac{2}{4}$	$9+2$	$6+1$	$5+2$	$9+8$	$7+1$
$14\frac{1}{4}$					

7.	8.	9.	10.	11.
hr. min.	hr. min.	da. hr.	sq. ft. sq. in.	A. sq. rd.
$3+20$	$9+30$	$12+18$	$12+100$	$8+120$
$4+50$	$8+20$	$6+20$	$30+120$	$12+100$

SUBTRACTION OF COMPOUND DENOMINATE NUMBERS

173. Subtract:

1.	2.	3.	4.
bu. pk.	ft. in.	gal. qt.	mi. rd.
$5+3$	$8+10$	$24+3$	$27+56$
$2+2$	$5+9$	$9+2$	$12+18$
$3+1$			

5. From 5 bu. 2 pk. subtract 2 bu. 3 pk.

$$\begin{array}{r}\text{bu. pk.}\\ 5+2\\ \underline{2+3}\\ 2+3\end{array}$$

3 pk. cannot be taken from 2 pk. Add to the 2 pk. the number of pecks it takes to make a unit of the next higher denomination, or add 4 pk. to 2 pk. The sum is 6 pk. 3 pk. from 6 pk. leaves 3 pk. We have added 1 bu. to the minuend, and must add 1 bu. to the subtrahend. 1 bu. + 2 bu. = 3 bu. 3 bu. from 5 bu. = 2 bu.

(If teacher prefers, she may regard the 1 bu. as taken from the 5 bu., leaving it 4 bu. 2 bu. from 4 bu. = 2 bu.)

To the Teacher. The work is somewhat shortened by following the same plan as was given on p. 199 for subtracting one mixed number from another. In applying it here, subtract 3 pk. from 1 bu. The remainder is 1 pk. Add the 1 pk. to 2 pk. and write the sum below. Subtract 3 bu. from 5 bu., or 2 bu. from 4 bu., and write the remainder, 2 bu., below.

6.	7.	8.	9.	10.
ft. in.	yd. ft.	hr. min.	lb. oz.	gal. qt.
7 + 6	12 + 1	27 + 15	12 + 10	24 + 2
2 + 9	7 + 2	10 + 40	9 + 12	9 + 3

11. From a piece of land containing $40\frac{1}{2}$ A., a man sells 10 A. and 40 sq. rd. How much has he left?

12. A piece of land is 40 rd. long and 20 rd. wide. How many square rods are there in the piece? How many acres? If $2\frac{1}{4}$ A. are sold, how much is left? Work in two ways.

13. How many square yards are there in a garden 36 ft. by 60 ft.? If a bed of lettuce 10 ft. wide extends across the garden, what is the area of the remaining portion of the garden? Represent the garden by a figure drawn on a scale of $\frac{1}{4}$ inch to the foot.

MULTIPLICATION OF COMPOUND DENOMINATE NUMBERS

174. To multiply compound denominate numbers.

RULE: *Beginning with the lowest, multiply the given number of each denomination by the multiplier. Reduce each product to higher denominations, when possible, and add units of the same denomination.*

Study Recitation

1. Multiply 2 bu. 3 pk. 7 qt. by 5.

What is the multiplier?

What is the number of the lowest denomination?

What is the product of this number by the multiplier?

What is the denomination of the second number to be multiplied?

What is the second product?

What is the third product?

$$\begin{array}{rrr} \text{bu.} & \text{pk.} & \text{qt.}\\ 2 & 3 & 7\\ & & 5\\ \hline 10 & 15 & 35\end{array}$$

What part of the rule has been applied to get this result?

How is the reduction made? $35\text{ qt.} = \overset{\text{bu.}}{1} + \overset{\text{pk.}}{0} + \overset{\text{qt.}}{3}$

How is the reduction made? $15\text{ pk.} = \overset{\text{bu.}}{3} + \overset{\text{pk.}}{3}$

What part of the rule has been applied to get these results?

$$\begin{array}{rl} & \text{bu.}\quad \text{pk.}\quad \text{qt.}\\ 10\text{ bu.} = & \underline{10\qquad\qquad\;}\\ 10\text{ bu. }15\text{ pk. }35\text{ qt.} = & 14+3+3\end{array}$$

What part of the rule has been applied to get the final result?

Multiply:

2.	3.	4.
pk. qt.	pk. qt.	pk. qt.
2 + 6	1 + 5	2 + 3
4	25	34

5.		**6.**		**7.**	
ft.	in.	ft.	in.	yd.	ft.
2+	9	1+	8	25+	2
	4		15		125

8. A man puts in each of 1 dozen bottles, 3 quarts 1 pint of water; how much does he put in the dozen bottles?

9. School is in session 6 hours a day. If pupils are in the schoolroom 5 hours and 25 minutes each day, how much time do they spend in the schoolroom in one week? How much of the time the school is in session are pupils not in the schoolroom during the week?

10. If a butcher sells meat at 15¢ a pound, how much does he receive from 3 customers, each of whom buys $5\frac{1}{4}$ lb.?

DIVISION OF COMPOUND DENOMINATE NUMBERS

175. To divide compound denominate numbers.

RULE: *Divide the number of units of the highest denomination in the dividend by the divisor; write the quotient above the partial dividend used; reduce the remainder, if any, to the next lower denomination, and add to this the number of this denomination, if any, in the dividend. Divide this sum, placing the quotient at the right of the partial quotient already found. Proceed as before with the remainder, if any. If the division is exact in any cases, proceed with the number of the next lower denomination, as at the beginning of the work.*

Study Recitation

1. Divide 35 yd. 1 ft. 6 in. by 6.

What is the number of the highest denomination?

On dividing 35 yd. by 6, what is the quotient and the remainder?

What is done with the remainder, 5 yd.?

On dividing 16 ft. by 6, what is the quotient and the remainder?

What is done with the remainder, 4 ft.?

On dividing 54 in. by 6, what is the quotient?

$$\begin{array}{rlll}
 & 5\text{ yd.}+ & 2\text{ ft.}+ & 9\text{ in.}\\ \cline{2-4}
6) & 35\text{ yd.} & 1\text{ ft.} & 6\text{ in.}\\
 & 30 & & \\ \cline{2-2}
 & 5\text{ yd.}= & 15\text{ ft.} & \\ \cline{3-3}
 & & 16\text{ ft.} & \\
 & & 12 & \\ \cline{3-3}
 & & 4\text{ ft.}= & 48\text{ in.}\\ \cline{3-4}
 & & & 54\text{ in.}\\
 & & & 54\\ \cline{4-4}
\end{array}$$

What part of the rule has been applied when the first remainder is found?

What more of the rule has been applied when you have found the next partial dividend, 16 ft.?

What more of the rule has been applied in completing the work?

Divide:

2. 4)14 bu. 3 pk.

3. 5)14 bu. 3 pk. 3 qt.

4. 4)25 yd. 2 ft.

5. 6)20 ft.

6. 8)16 lb. 8 oz.

7. 5)12 mi. 45 rd.

8. A man rows 20 mi. 20 rd. in 3 hr. How far does he row in 1 hr.?

9. Ten yards of ribbon will make how many badges each 5 in. long? If the ribbon is made into 60 badges, how long will each badge be?

10. A daily ration of 24 lb. of bread is divided among 18 men. What is each one's share?

PERCENTAGE

Study Recitation

176. 1. What is $\frac{1}{2}$ of 24? $\frac{1}{4}$ of 48? $\frac{2}{5}$ of 35? $\frac{3}{4}$ of 32?

Finding each result is finding a fractional part of a number.

2. What is $\frac{50}{100}$ of 24? $\frac{25}{100}$ of 48? $\frac{40}{100}$ of 35? $\frac{75}{100}$ of 32?

Finding each result is finding a number of hundredths of a number. Notice that the fractions in 1 are reduced to hundredths in each case in 2.

3. What is .5 of 24? .25 of 48? .4 of 35? .75 of 32?

Finding each result is finding a decimal part of a number. Notice that the fractions in 1 are changed to decimals in 3, and that these decimals are the equals of the fractions in 2. Thus, $.5 = \frac{5}{10}$; $.25 = \frac{25}{100}$, etc.

$$\frac{1}{2} = \frac{50}{100} = .50 = .5$$
$$\frac{1}{4} = \frac{25}{100} = .25$$
$$\frac{2}{5} = \frac{40}{100} = .40 = .4$$
$$\frac{3}{4} = \frac{75}{100} = .75$$

Per cent means hundredths.

5 per cent means $\frac{5}{100}$, or $\frac{1}{20}$, or the decimal .05
7 per cent means $\frac{7}{100}$, or the decimal .07
25 per cent means $\frac{25}{100}$, or $\frac{1}{4}$, or the decimal .25
30 per cent means $\frac{30}{100}$, or $\frac{3}{10}$, or the decimal .30

The sign of per cent is %.

To find any per cent of a number is to find that number of hundredths of the number, using the hundredths as a common fraction, or in its lowest terms, or as a decimal expressing hundredths.

1. Find 25 % of 32.

To find 25 % of 32 is the same as to find

$$\frac{25}{100} \text{ or } \frac{1}{4}, \text{ or } .25, \text{ of } 32.$$

$$\frac{\cancel{25}}{\underset{4}{\cancel{100}}} \text{ of } 32 = 8. \qquad \frac{1}{4} \text{ of } 32 = 8. \qquad \begin{array}{r} .32 \\ \underline{.25} \\ 160 \\ \underline{64} \\ 8.00 \end{array}$$

177. TABLE OF PER CENTS AND EQUIVALENT FRACTIONS AND DECIMALS

$5\% = \frac{5}{100} = \frac{1}{20} = .05$

$10\% = \frac{10}{100} = \frac{1}{10} = .10$ or $.1$

$20\% = \frac{20}{100} = \frac{1}{5} = .20$ or $.2$

$25\% = \frac{1}{4} = .25$

$50\% = \frac{1}{2} = .50$ or $.5$

$75\% = \frac{3}{4} = .75$

$40\% = \frac{2}{5} = .40$ or $.4$

$60\% = \frac{3}{5} = .60$ or $.6$

$$\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = 33\tfrac{1}{3}\,\% = .33\tfrac{1}{3}$$

$$\tfrac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3}\,\% = .66\tfrac{2}{3}$$

$$\tfrac{1}{8}=\frac{12\frac{1}{2}}{100}=12\tfrac{1}{2}\%=.12\tfrac{1}{2} \text{ or } .125$$

$$\tfrac{3}{8}=\frac{37\frac{1}{2}}{100}=37\tfrac{1}{2}\%=.37\tfrac{1}{2} \text{ or } .375$$

$$\tfrac{5}{8}=\frac{62\frac{1}{2}}{100}=62\tfrac{1}{2}\%=.62\tfrac{1}{2} \text{ or } .625$$

$$\tfrac{7}{8}=\frac{87\frac{1}{2}}{100}=87\tfrac{1}{2}\%=.87\tfrac{1}{2} \text{ or } .875$$

178. DRILL

Tell what per cents correspond to the fractions given below, and what fractions correspond to the per cents or decimals given below.

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$.
2. 25%, 10%, 20%, 30%, 75%, 80%, 60%.
3. $\frac{2}{3}$, $\frac{4}{5}$, 40%, .8, .75, .60, .3.
4. $\frac{7}{8}$, $\frac{1}{2}$, $\frac{3}{10}$, $62\frac{1}{2}\%$, $37\frac{1}{2}\%$, $66\frac{2}{3}\%$, $87\frac{1}{2}\%$.
5. Find $\frac{1}{2}$ of each of the following numbers:
   12, 18, 25, 30, 62, 48, 256, 312.

6. What per cent of the numbers in example 5 have you found in each case?

7. Prove your work to be correct by multiplying each number by the decimal .50. Multiply each number by the decimal .5, and compare the results.

8. Find .25 of each of the following numbers by the process of multiplication by the decimal:

16, 24, 60, 180, 254, 82, 96, 480, 320, 1600, 5600, 3840.

9. What per cent of each number have you found? What part of each number? Prove your work by finding this part of each number.

10. 7 % of a number equals what part of it? What decimal of it? Find $\frac{7}{100}$ of 365. Find .07 of 365 by multiplication. Which is the better way?

11. Find 25 % of 365, using the per cent as a fraction in its lowest terms.

Find 25 % of 365 by multiplying by the decimal .25. Which is the better way? Why?

179. To find any per cent of a number.

Study Recitation

1. Find 10 % of 30.

This is a special case of the general problem in common fractions: To find a fractional part of a number.

2. Find $\frac{1}{10}$ of 30.

Every example in which it is required to find a per cent of a number, may be changed to an example in common fractions, in which it is required to find a fractional part of a number. Review exercise 133, p. 220, before proceeding further.

3. Find $37\frac{1}{2}$ % of 24.

When changed this becomes: Find $\frac{3}{8}$ of 24. It may also be changed to an example in multiplication of decimals.

ILLUSTRATION: Find $37\frac{1}{2}$ % of 24. Changed, becomes: Find .375 of 24, or $24 \times .375 = ?$ $37\frac{1}{2}$ % of $24 = \frac{3}{8}$ of $24 = .375 \times 24$.

Change the following examples in percentage to examples in common fractions, and to examples in multiplication of decimals. Fill blanks, and solve:

	Percentage	Common Fractions	Multiplication of Decimals
4.	Find 75% of 48.	Find $\frac{3}{4}$ of 48.	Find .75 of 48.
5.	Find $66\frac{2}{3}\%$ of 21.	Find —— of 21.	Find —— of 21.
6.	Find 10% of 40.	Find —— of 40.	Find —— of 40.
7.	Find 10% of 25.	Find —— of 25.	Find —— of 25.
8.	Find 25% of 18.	Find —— of 18.	Find —— of 18.
9.	Find 40% of 30.	Find —— of 30.	Find —— of 30.

To find any per cent of a number.

RULE I: *Reduce the given per cent to a fraction in its lowest terms; find this fractional part of the number. Or,*

RULE II: *Multiply the number by the given per cent expressed as a decimal.*

ORAL

Use Rule I

10. Find 5% of 40; 60; 80; 85; 90; 120.
11. Find 4% of 50; 100; 200; 75; 125; 150.
12. Find $12\frac{1}{2}\%$ of 24; 32; 42; 64; 80; 84.
13. Find $33\frac{1}{3}\%$ of 18; 9; 12; 14; 38; 45.
14. Find $66\frac{2}{3}\%$ of 21; 12; 15; 30; 24; 27.
15. Find 20% of 25; 10; 14; 18; 35; 125.
16. Find 75% of 24; 8; 12; 14; 16; 29.

WRITTEN

Use Rule II

17. Find 7% of 28; 39; 251; 175.

18. Find 12% of 36; 49; 87; 120.

19. Find 6% of 18; 36; 45.6; 27.3.

20. Find 8% of 2.7; 3.5; 2.78; 15.6.

21. Find 3% of 150; 48; $273\frac{1}{3}$; $28\frac{2}{3}$.

22. Write as decimals: $3\frac{1}{2}\%$; $7\frac{1}{2}\%$; $8\frac{1}{4}\%$.

23. Find $3\frac{1}{2}\%$ of 48; 24; 36; 125; 27.

24. Find $7\frac{1}{2}\%$ of 25; 40; 20; 35; 426.

Work by both rules and decide which one is the better:

25. Find $12\frac{1}{2}\%$ of 24; 56; 83; 128; 250.

26. Find 25% of 16; 28; 34; 46; 142.

27. Find 20% of 15; 35; 36; 48; 27.

28. Find 75% of .28; 18; 19; 42; 126.

PROBLEMS

180. ORAL AND WRITTEN

1. From a tank containing 50 gal. of vinegar, 10% leaked out. How many gallons were lost? How many were left in the cask?

ANALYSIS: 10% of a number is $\frac{1}{10}$ of it; $\frac{1}{10}$ of 50 gal. is 5 gal., which leaked out. There were left 50 gal. less 5 gal., or 45 gal.

2. In an orchard there are 75 trees. If 20% of them are plum trees, and the others are apple trees, how many trees of each kind are there in the orchard?

3. A merchant bought cloth at 8¢ a yard, and sold it for 25% more than it cost. For how much more than the cost did he sell it? What was the selling price per yard? If he sold it so as to lose 25%, how much less than the cost did he receive? How much did he sell it for per yard?

4. If a clothing dealer pays $15 apiece for overcoats, and sells them at a gain of 20%, what is the selling price? How much does he make on one coat? On 1 doz. coats?

5. A farmer tests 80 kernels of corn to see how many will grow. 12½% fail to grow. How many kernels grow? How many fail to grow?

6. There are 40 words in a spelling lesson. If a boy misses 10% of them, how many does he spell correctly?

7. If I fail to work 5% of the problems in my arithmetic lesson, and there are 20 problems in the lesson, how many do I work? What per cent of them do I work? How many do I fail to work?

8. A farmer harvested 250 bushels of potatoes, but 10% of them were spoiled by freezing. How many bushels were spoiled? How many bushels were not damaged?

9. If a man buys a horse for $88, and sells it so as to gain 12½%, for how much does he sell the horse? How much does he gain?

10. A boy lost 8 marbles, which was $\frac{1}{8}$ of what he had at first. How many had he at first?

11. Change problem 10 to a problem in percentage, and work it.

181. To find what per cent one number is of another.

1. 6 is what per cent of 8?

This is a special case of the general problem in fractions: To find what fractional part one number is of another.

2. 6 is what part of 8?

The fractional part may be found by the rule given in exercise 136, p. 223. The fraction must then be changed to hundredths.

Change the following examples in percentage to examples in common fractions. After the changes have been made, give the answers in fractions in lowest terms.

Give the equivalents of the fraction in per cents.

ILLUSTRATION: 8 is what % of 64? Changed, 8 is what fraction of 64? Answer, $\frac{1}{8}$ of 64. $\frac{1}{8}$ of 64 $= 12\frac{1}{2}\%$, or $.12\frac{1}{2}$, of 64.

3. 2 is what % of 8?

4. 9 is what % of 12?

5. 8 is what % of 10?

6. 8 is what % of 12?

7. 12 is what % of 18?

8. 6 is what % of 8?

9. 6 is what % of 9?

10. 6 is what % of 10?

11. 6 is what % of 12?

12. 6 is what % of 15?

13. 6 is what % of 18?

14. 6 is what % of 24?

15. 8 is what % of 24?

16. 8 is what % of 20?

17. 10 is what % of 25?

18. 20 is what % of 80?

19. 20 is what % of 40?

20. 20 is what % of 25?

21. 15 is what % of 20?

22. 18 is what % of 24?

23. A man sold $\frac{1}{3}$ of his farm; what per cent of it did he sell? How many hundredths did he sell?

How many hundredths of the farm had he left?

What per cent of the farm had he left?

24. A pupil solves correctly $\frac{3}{4}$ of the problems in his lesson; what part of the problems in his lesson are still to be solved? What per cent of the problems has he solved correctly? What per cent of the problems has he still to solve?

25. In a field of 20 acres there are 15 acres of corn; what per cent of the field is corn?

26. From a pile of wood containing 24 cd. 20 cd. are sold; what per cent of the wood is sold? What per cent remains unsold?

27. A man has 30 cords of wood and sells 10% of it at $4 a cord; how much does he receive for what he sells? How much is the rest of the wood worth at the same rate?

The next two problems are unlike any thus far given. Can you solve them?

28. 8 is 25% of what number?

29. A man pays $20 for a suit of clothes; the cost of the clothes is 50 % of what he earns in 1 month. How much does he earn in 1 month?

The per cent one number is of another may be found in another way.

30. 3 is what per cent of 4? The first step is the same as in the preceding examples. 3 is what fraction of 4? *Ans.* $\frac{3}{4}$. $\frac{3}{4}$ is the same as $3 \div 4$.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \end{array}$$

To find how many hundredths, or what per cent 3 is of 4, place a decimal point after 3, annex two zeros, and divide as in division of decimals. The result is .75 or 75 %.

31. 6 is what per cent of 8? 6 is what fraction of 8? 6 is what part of 8?

These three questions ask for the same thing. 6 is a part of 8; then in this case 8 is to be thought of as the whole and 6 as the part.

In examples like 31, the number following the word "of" is the denominator of the fraction of which the other number is the numerator.

To find what per cent one number is of another.

SECOND RULE: *Divide the number expressing the part, by the number expressing the whole, continuing the division to hundredths' place.*

32. 8 is what % of 12?

8 is the part. 12 is the whole.
Applying the rule, we have

$$\begin{array}{r} .66\frac{2}{3} = 66\frac{2}{3}\,\% \\ \text{Whole } 12\overline{)8.00} \text{ Part} \end{array}$$

33. 5 is what % of 6? Applying the rule $\begin{array}{l} \quad .83\frac{1}{3} = 83\frac{1}{3}\% \\ 6\overline{)5.00} \end{array}$

34. 9 is what % of 10?

35. 5 is what % of 7?

36. 8 is what % of 9?

37. 12 is what % of 14?

38. 10 is what % of 50?

39. 14 is what % of 28?

40. 6 is what % of 8?

41. 7 is what % of 42?

42. 15 is what % of 18?

43. 25 is what % of 50?

44. 25 is what % of 80?

45. 30 is what % of 80?

How is a common fraction changed to a decimal?

The second rule is used when the per cent which the fraction equals is not readily seen.

182. To find a number when a certain per cent of it is given.

Study Recitation

1. 12 is 20% of what number?

This is a special case of the general problem in fractions: To find a number when a fractional part of it is given.

Review exercise 138, p. 224.

2. 12 is $\frac{1}{5}$ of what number?

3. 18 is $\frac{2}{5}$ of what number?

Analysis: If 18 is $\frac{2}{5}$ of some number, $\frac{1}{5}$ of that number is $\frac{1}{2}$ of 18, or 9. $\frac{5}{5}$ of the number = 5 times 9, or 45.

18 is 40% of what number? This is the same question as "18 is $\frac{2}{5}$ of what number?" because 40% of a number is $\frac{2}{5}$ of it.

Change the following examples in percentage to examples in common fractions; then solve them.

Percentage	Common Fractions
4. 8 is 25% of what number?	Changed, becomes 8 is $\frac{1}{4}$ of what number?
5. 24 is $33\frac{1}{3}\%$ of what number?	Changed, becomes 24 is $\frac{1}{3}$ of what number?
6. 12 is $66\frac{2}{3}\%$ of what number?	Changed, becomes 12 is $\frac{2}{3}$ of what number?
7. 35 is 50% of what number?	Changed, becomes............
8. 16 is 80% of what number?	Changed, becomes............
9. $5\frac{1}{2}$ is 25% of what number?	Changed, becomes............
10. 7 is 20% of what number?	Changed, becomes............
11. 9 is 75% of what number?	Changed, becomes............

When a certain per cent of a number is given, the number may also be found without reducing the per cent to a fraction.

12. 21 is 7% of what number?

Analysis: If 21 is 7% of a number, 1% of the number is $\frac{1}{7}$ of 21, or 3. 100% of the number is 100 times 3, or 300.

Operation: $21 \div 7 = 3$. $100 \times 3 = 300$, or

$$\begin{array}{r} 300 \\ {}_{|}07.\overline{)21_{|}00.} \end{array}$$

To find a number when a certain per cent of it is given.

1st Rule: *Divide the number expressing the part by the rate per cent used as a whole number, and multiply the quotient by* 100, or

2d Rule: *Divide the number expressing the part by the rate per cent used as a decimal.*

The second rule is more convenient when the number expressing the rate per cent will not exactly divide the number expressing the part.

13. 25 is 8 % of what number?

Operation by First Rule: Operation by Second Rule:

$$8\overline{)25} \qquad \underline{3\tfrac{1}{8}}$$

$$\begin{array}{r} 100 \\ \underline{3\frac{1}{8}} \\ 12\frac{1}{2} \\ \underline{300\phantom{\frac{1}{2}}} \\ 312\frac{1}{2} \text{ or} \\ 312.5 \end{array}$$

$$\begin{array}{r} 312.5 \\ {}_{\wedge}08.\overline{)25_{\wedge}00.0} \end{array}$$

Analyze:

14. 24 is 12% of what number?

Analysis: If 24 is 12 % of a number, 1 % of the number is $\frac{1}{12}$ of 24, or 2. 100 % of the number is 100 × 2, or 200.

15. 28 is 7 % of what number?

16. 18 is 9 % of what number?

17. 24 is 6 % of what number?

18. 22 is 11 % of what number?

19. 30 is 15 % of what number?

20. 21 is 3 % of what number?

21. 25 is 5 % of what number?

The above analysis requires the application of the first rule.

Solve the following examples by the second rule:

22. 15 is 7 % of what number?

OPERATION:

$$\begin{array}{r} 214\frac{2}{7}. \\ .07.\overline{)15.00.} \end{array}$$

23. 19 is 7 % of what number?

24. 256 is 16 % of what number?

25. 250 is 12 % of what number?

26. 12.5 is 4 % of what number?

27. 3.25 is 6 % of what number?

28. 46.8 is 8 % of what number?

29. Make and solve examples in two ways. (1) where it is required to find some per cent of a number; (2) where it is required to find what per cent one number is of another; (3) where it is required to find a number when a certain per cent of it is given.

30. Give the rules: (1) for finding any per cent of a number; (2) for finding what per cent one number is of another; (3) for finding a number when a certain per cent of it is given.

INTEREST

183. **Study Recitation**

One man sometimes rents or hires a piece of land from another, and agrees to pay a certain sum, called rent, per year for its use. Or he may borrow or hire money and agree to pay a certain per cent of it for its use for one year.

A borrows $250 from B for one year, and agrees to pay B the $250 at the end of the year, and also a sum equal to 6 % of the $250 borrowed. If A borrows the money for more than one year, he pays 6 % of it each year for its use, and for any fraction of a year he pays that fraction of 6 % of $250. He may pay 5 %, or any other rate, for the use of the money, as is agreed.

The **principal** is the sum of money borrowed.

The **interest** is the sum paid for the use of the principal.

The **rate** is the per cent of the principal paid for its use for one year.

The **amount** is the sum of the principal and the interest.

Study Recitation

1. A borrows $250 of B for one year at 6 %. What is the interest? The amount?

OPERATION:

$250 Principal
.06 Rate
15.00 Int. for 1 yr.
250.00
265.00 Amount

What is the interest for 1 yr.? What is the interest for 2 years? For 4 years? What is the interest for $\frac{1}{2}$ year, or 6 months? What is the interest for $\frac{1}{3}$ year, or 4 months? What is the interest for $\frac{1}{12}$ year, or 1 month?

2. C borrows $120 of D for 2 years 6 months, at 8%. What is the interest? The amount?

OPERATION:

$120	Principal
.08	Rate
9.60	Interest for 1 year
$2\frac{1}{2}$	No. of years
4.80	Interest for $\frac{1}{2}$ year, or 6 months
19.20	Interest for 2 years
24.00	Interest for 2 years 6 months
120.00	Principal
144.00	Amount due at end of 2 years 6 months

Find the interest and amount for the given rate and time:

3. $175 for 3 years, at 7%.
4. $125 for 2 years, at 6%.
5. $280 for 1 year 6 months, at 6%.
6. $560 for 4 years 4 months, at 8%.
7. $150.50 for 3 years 2 months, at 6%.
8. $240 for 1 year 1 month, at 6%.
9. $240 for 1 year 5 months at 6%.
10. $240 for 2 years 5 months, at 6%.

In computing interest, 30 days are usually regarded as a month. Since there are 30 days in 1 month, in $\frac{1}{10}$ of a month there are 3 days.

Any number of days may be reduced to tenths of a month by dividing by 3.

RULE: *To reduce months and days to months and decimal of a month, divide the number of days by 3, and annex the quotient as tenths to the number of months.*

11. Reduce 4 months 27 days to months and decimal of a month.

Divide 27 by 3, and annex the quotient 9 as .9 to the 4; thus 4.9 months.

12. Reduce 5 months 12 days to months and decimal of a month.

$12 \div 3 = 4$. Annex 4 as .4 to 5; thus, 5.4 months.

13. Reduce 6 months and 14 days to months and decimal of a month.

$14 \div 3 = 4\frac{2}{3}$. Annex $4\frac{2}{3}$ as $.4\frac{2}{3}$ to 6; thus, $6.4\frac{2}{3}$ months.

Fill the blanks in the following:

14. 5 months 24 days = —— months.
15. 6 months 21 days = —— months.
16. 7 months 13 days = —— months.
17. 8 months 10 days = —— months.
18. 9 months 4 days = —— months.
19. 3 months 5 days = —— months.
20. 2 months 7 days = —— months.
21. 4 months 11 days = —— months.
22. 5 months 16 days = —— months.
23. 3 months 22 days = —— months.

Study Recitation

24. What is the interest of $ 200 for 2 years 5 months 15 days, at 6 %?

Operation:

$ 200 5 mo. 15 da. = 5.5 mo.
.06
12)$ 12.00 × 2 = $ 24.00 Int. for 2 yr.
Int. for 1 mo. $ 1.00 5.50 Int. for 5 mo. 15 da.
5.5
.500 $ 29.50 Total int.
500
Int. for 5 mo. $ 5.500
15 da.

25. What is the interest on $ 225 for 3 years 7 months 14 days, at 8 %? The amount?

Operation:

$ 225 7 mo. 14 da. = $7.4\frac{2}{3}$ mo.
.08
12)$ 18.00 × 3 = $ 54.00 Int. for 3 yr.
Int. for 1 mo. $ 1.50 11.20 Int. for 7 mo. 14 da.
$7.4\frac{2}{3}$
100
600 $ 65.20 Total int.
10.50 225.00 Principal
Int. for 7 mo. $ 11.200 $ 290.20 Amount
14 da.

What is the interest and amount on each of the following at 6%; at 7%; at 8%; at 10%:

26. $180 for 3 yr. 9 mo. 18 da.

27. $165 for 2 yr. 7 mo. 9 da.

28. $224 for 6 yr. 2 mo. 4 da.

29. $168 for 2 yr. 11 mo. 6 da.

30. $250 for 2 years 5 months 12 days, at 7%?

OPERATION:

$250 5 mo. 12 da. = 5.4 mo.
.07
12)$17.50 × 2 = $35.00 Int. for 2 yr.
Int. for 1 mo. $\$1.458^{+}$ 7.87 Int. for 5 mo. 12 da.
5.4 $42.87 Total int.
5832 250.00 Principal
7290 $292.87 Amount
$7.8732 Int. for 5 mo. 12 da.

STATEMENT OF OPERATION IN GENERAL TERMS

Multiply the principal, ——, by ——, the rate expressed decimally; the product is ——, the interest for 1 year.

Multiply ——, the interest for 1 year, by ——, the number of years; the product, ——, is the interest for ——, the given number of years.

Divide ——, the interest for 1 year, by 12; the quotient, ——, is the interest for 1 month.

Reduce the —— days to 10ths of a month, and annex to ——, the number of months. Multiply ——, the interest for 1 month, by ——, the number expressing the months and decimal of a month; the product, ——, is the interest for ——, the given number of months and days. Add this product, ——, to ——, the interest for the given number of years; this sum, ——, is the total interest.

TO THE TEACHER. Require pupils to give the above statement of the process, filling in the blanks with the correct numbers in each case. If the pupils give the general statement, ignoring the blanks, the rule is given.

184. To find the difference in time between two dates.

1. What is the difference in time between May 12, 1907 and Jan. 2, 1909?

Write the dates for subtraction, writing the year, the number of the month, and the day of the month in each case:

$$\begin{array}{lr}
 & \text{yr.} \quad \text{mo.} \quad \text{da.} \\
\text{Jan. 2, 1909, is written} & 1909 + 1 + \ \ 2 \\
\text{May 12, 1907, is written} & \underline{1907 + 5 + 12} \\
\text{Difference in time} & 1 + 7 + 20
\end{array}$$

As 12 cannot be taken from 2, add 1 month reduced to days, or 30 days, to 2, making 32 days. 12 from 32 = 20.

Add 1 month to 5 months, making 6 months. As 6 cannot be taken from 1, add 1 year, or 12 months, to 1, making 13. 6 from 13 = 7. Add 1 year to 1907, making 1908. 1908 from 1909 = 1.

Learn the number of each month:

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	2	3	4	5	6	7	8	9	10	11	12

2. What is the time from Sept. 15, 1896 to Dec. 12, 1900?

3. What is the time from June 5, 1898 to May 4, 1902?

4. How many years, months, and days is it from Aug. 6, 1901 to Oct. 3, 1905?

5. A man borrowed $240, March 15, 1903, at 6%; he paid it Jan. 12, 1905. What was the interest? The amount?

6. Aug. 10, 1890, a man loaned $150, at 8%. The note was paid June 4, 1892. What sum was paid?

ANSWERS

P. 78, Ex. 27. — **3.** 154. **4.** 146. **5.** 1614. **6.** 1648. **7.** 2027.

P. 79. — **10.** 131 mi. **11.** 102 desks. **12.** 112 lb. **13.** 82 ¢. **14.** 1514. **15.** 2587. **16.** 2112. **17.** 1796. **18.** 1669.

P. 82, Ex. 28. — **20.** 42 pupils. **22.** 91 da. **23.** 90 da. **24.** 92 da. **25.** 92 da.

P. 83. — **26.** 189 da. **27.** 29 ¢. **28.** 21. **33.** 38 yr. **34.** 138. **35.** $ 18.

P. 84, Ex. 29. — **1.** $268.27. **2.** $336.20. **3.** 81 ¢. **4.** $ 14.42. **5.** $39.08. **6.** $ 10.15, $ 5.60.

P. 94, Ex. 34. — **23.** 727. **24.** 1477. **25.** 1666. **26.** $ 14.99. **27.** $ 121.38.

P. 96. — **48.** 591. **49.** 445. **50.** 211. **51.** $ 2.64. **52.** $ 15.56. **53.** 46.46.

P. 98. — **65.** 396. **66.** 501. **67.** 500. **68.** 585. **69.** 559.

P. 115, Ex. 46. — **17.** 1054 sheep, Ans. to last. **18.** 113⅓ cords, Ans. to last. **19.** $ 12.46, $ 5.34. **20.** 64. **21.** 12 pencils. **22.** 9 ¢.

P. 118, Ex. 50. — **9.** $ 1160. **10.** $ 54. **11.** 160 mi. **13.** 33 weeks. **14.** $ 15, $ 24, $ 39. **15.** $ 375. **16.** 14186. **17.** 178. **18.** 1208. **19.** 2202. **20.** 2730. **21.** 2381.

P. 123, Ex. 53. — **27.** $ 1.68. **28.** $ 234, $ 78. **29.** $ 1725.30. **30.** $ 1950.

P. 124, Ex. 54. — **2.** 6144. **3.** 10464. **4.** 5084. **5.** 8080. **6.** 18225. **7.** 17664.

P. 124, Ex. 55. — **1.** $ 52.50. **2.** $ 1.44. **3.** $ 63, Ans. to last.

P. 125. — **4.** Arith. $ 12.60, Spellers $ 7.20, Readers $ 20.70, all $ 40.50. **6.** $ 10.65. **8.** $ 64 per mo., $ 16 per wk. **9.** $ 4474.

P. 126. — **11.** 8 pupils. **12.** 36, 216. **13.** 50. **14.** 5 weeks.

P. 132, Ex. 61. — **6.** 12 hours. **7.** 6 wk. **8.** 9509. **9.** 29184. **10.** 24550. **11.** 29799. **12.** 32 doz. **13.** 64 lb.

P. 136, Ex. 65. — **1.** 331. **2.** $772\frac{1}{7}$. **3.** $413\frac{8}{9}$. **4.** $7113\frac{5}{12}$. **5.** $253\frac{2}{11}$. **6.** $54.25.

P. 137. — **7.** $1400 for 7 carloads. **8.** $33.12

P. 142, Ex. 71. — **1.** $1308. **2.** $10441. **4.** 2125 mi. **5.** $9.

P. 143. — **6.** 1044 ft., Ans. to last. **7.** 69. **8.** 12¢. **10.** $44.10. **11.** $261.

P. 154, Ex. 81. — **1.** 2816. **2.** 4244. **3.** 5192. **4.** 102146. **5.** 52272. **6.** 37422.

P. 155. — **7.** 72, 79. **8.** 3288, 4272, 7188, 8208. **9.** 72, 31. **10.** 216, 93, 249. **11.** $116 more; $126, cost of horse; $34, cost of cow. **15.** 56. **16.** 24. **17.** 77. **19.** 1632 bu.

P. 158, Ex. 85. — **1.** 29472, 104676, 123048, 68136, 87528. **3.** 23 yr. **4.** $6. **5.** $936, $64. **6.** $50. **7.** $1.52. **8.** $110. **9.** 18 doz.

P. 164, Ex. 89. — **4.** 42. **5.** 7. **6.** 38.

P. 165. — **7.** 147. **8.** 23. **10.** 62. **11.** 54. **12.** 26. **13.** 45. **14.** 16. **15.** 65. **16.** 35. **17.** 63. **18.** 42. **19.** 23. **20.** 48. **21.** 57. **22.** 69. **23.** 46. **24.** $81\frac{14}{24}$. **25.** 76. **26.** 28. **27.** 69. **28.** 84. **29.** 81. **30.** 75. **31.** 98.

P. 168, Ex. 91. — **2.** 204. **3.** 803. **4.** 706. **5.** 507. **6.** 1005. **7.** 201. **8.** 1007. **9.** 203. **10.** 304.

P. 169, Ex. 92. — **2.** $250\frac{4}{43}$. **3.** 230. **4.** $1056\frac{7}{28}$. **5.** 13020. **6.** $10230\frac{5}{29}$. **7.** $3300\frac{7}{35}$.

P. 170, Ex. 94. — **3.** $117\frac{16}{20}$. **4.** $74\frac{18}{20}$. **5.** $178\frac{12}{20}$. **6.** $61\frac{18}{30}$. **7.** $116\frac{12}{40}$. **8.** $157\frac{25}{50}$. **9.** $21\frac{43}{120}$. **10.** $28\frac{124}{130}$. **11.** $32\frac{88}{140}$.

P. 172, Ex. 95. — **3.** 593. **4.** 869. **5.** 879. **6.** $19\frac{206}{426}$. **7.** $12\frac{49}{523}$. **8.** 4.

P. 172, Ex. 96. — **1.** 216. **2.** 384. **3.** 762. **4.** 453. **5.** $1243\frac{18}{23}$. **6.** $55\frac{59}{62}$. **7.** $41\frac{78}{83}$. **8.** $249\frac{41}{83}$. **9.** $294\frac{20}{92}$. **10.** $77\frac{68}{74}$. **11.** 7426. **12.** 2454. **13.** 916. **14.** 7937. **15.** 834. **16.** 888. **17.** 541. **18.** 723. **19.** $518\frac{32}{37}$. **20.** $251\frac{16}{54}$. **21.** $832\frac{18}{43}$. **22.** $1282\frac{15}{28}$. **23.** 238. **24.** 506. **25.** 27. **26.** $148\frac{30}{98}$. **27.** 261. **28.** 463. **29.** 207. **30.** 463.

P. 175, Ex. 98. — **1.** $42, $5.25. **2.** $5200. **3.** $32, $4. **5.** $4745. **6.** 85¢. **7.** $1.40.

P. 176. — **8.** $2.80. **9.** $1.60, Ans. to last. **10.** $4. **11.** 240 rods. **12.** $600. **13.** 282 bbl., 47 bbl. **14.** $5200. **15.** 45. **16.** 780.

P. 177. — **17.** 15. **18.** $10050, Ans. to last. **19.** 2604 mi. **20.** $1000. **24.** 3414. **25.** $132.72. **26.** $25.20. **27.** $298.64.

P. 178. — **28.** $36.

P. 197, Ex. 114. — **3.** $1\frac{7}{15}$. **4.** $1\frac{1}{24}$. **5.** $1\frac{1}{6}$. **6.** $1\frac{1}{10}$. **7.** $1\frac{11}{24}$. **8.** $1\frac{11}{12}$. **9.** $1\frac{27}{28}$. **10.** $1\frac{1}{40}$. **11.** $1\frac{1}{7}$. **12.** $\frac{5}{24}$. **13.** $\frac{1}{16}$. **14.** $\frac{1}{8}$. **15.** $\frac{8}{45}$. **16.** $\frac{2}{15}$. **17.** $\frac{1}{4}$. **18.** $\frac{5}{18}$. **19.** $\frac{7}{30}$. **20.** $\frac{5}{14}$.

P. 198, Ex. 115. — **2.** $15\frac{1}{4}$. **3.** $10\frac{1}{2}$. **4.** $20\frac{3}{10}$. **5.** $27\frac{19}{21}$. **6.** $27\frac{1}{24}$. **7.** $7\frac{7}{18}$. **8.** $18\frac{2}{15}$. **9.** $29\frac{5}{8}$. **10.** $27\frac{4}{9}$. **11.** $23\frac{5}{24}$. **13.** $29\frac{9}{20}$. **14.** 30. **15.** $20\frac{1}{2}$. **16.** $18\frac{3}{5}$.

P. 199, Ex. 116. — **3.** $3\frac{7}{15}$. **4.** $8\frac{17}{56}$. **5.** $11\frac{2}{5}$. **6.** $11\frac{7}{8}$. **7.** $15\frac{13}{15}$. **8.** $14\frac{13}{14}$. **9.** $2\frac{13}{24}$. **10.** $9\frac{11}{24}$. **11.** $14\frac{2}{15}$.

P. 199, Ex. 117. — **1.** 25¢. **2.** $8\frac{11}{12}$ mi., $3\frac{1}{12}$ mi. **3.** \$$26\frac{1}{4}$. **4.** $11\frac{1}{4}$ gal.

P. 200. — **5.** $2\frac{2}{3}$ hrs. **6.** 2 books, 50¢. **7.** $33\frac{5}{12}$. **8.** $38\frac{1}{24}$. **9.** $69\frac{4}{5}$. **10.** 84. **11.** $69\frac{1}{3}$.

P. 200, Ex. 118. — **1.** $72\frac{11}{12}$. **2.** $3528\frac{1}{8}$. **3.** $4\frac{3}{8}$. **4.** $49\frac{2}{5}$. **5.** $18\frac{1}{6}$. **6.** $10\frac{5}{8}$ gal. **7.** $\frac{2}{5}$.

P. 201. — **8.** $\frac{5}{16}$. **9.** $\frac{5}{24}$. **10.** $24\frac{3}{8}$ lb. **11.** $7\frac{1}{2}$ mi. **12.** $8\frac{3}{4}$ lb. **13.** $62\frac{1}{2}$ in. **14.** $1\frac{11}{12}$ yd. **15.** 655, $13\frac{1}{24}$.

P. 202. — **17.** $4\frac{5}{8}$ in., Ans. to last. **18.** 10008, 29775. **21.** 146612.

P. 207, Ex. 122. — **3.** $1\frac{1}{15}$. **4.** $2\frac{4}{5}$. **5.** $2\frac{11}{182}$. **6.** $\frac{2662}{3375}$. **7.** $\frac{1}{16}$. **8.** 60.

P. 208. — **10.** $\frac{1}{3}$. **11.** $\frac{15}{64}$. **12.** $\frac{9}{20}$. **13.** $31\frac{1}{9}$. **14.** 56. **15.** $33\frac{1}{24}$. **18.** $30, Ans. to last.

P. 212, Ex. 126. — **1.** 21. **2.** $30\frac{1}{3}$. **3.** $11\frac{7}{8}$. **4.** $15\frac{1}{2}$. **5.** $16\frac{1}{2}$. **6.** $29\frac{1}{3}$. **7.** 45. **8.** $57\frac{2}{5}$. **9.** $21\frac{1}{2}$. **10.** $47\frac{1}{2}$. **11.** 24. **12.** 102. **13.** $95\frac{1}{5}$. **14.** $122\frac{5}{8}$. **15**: $97\frac{1}{2}$.

P. 213, Ex. 128. — **1.** 11. **2.** $6\frac{9}{16}$. **3.** $24\frac{2}{3}$. **4.** $232\frac{2}{5}$. **5.** $130\frac{1}{5}$. **6.** $11\frac{2}{3}$. **7.** $26\frac{4}{9}$. **8.** $11\frac{2}{3}$. **9.** $67\frac{3}{16}$.

P. 215, Ex. 130. — **1.** $1\frac{1}{8}$. **2.** $\frac{16}{33}$. **3.** 22. **4.** $10\frac{1}{3}$. **5.** $2\frac{2}{3}$. **6.** $\frac{10}{21}$. **7.** $2\frac{2}{5}$. **8.** 34. **9.** 22. **10.** $32\frac{14}{15}$.

P. 216. — 11. $2\frac{7}{24}$. **12.** $\frac{25}{42}$. **13.** $\frac{8}{25}$. **14.** 20. **15.** $2\frac{3}{5}$. **16.** $1\frac{1}{20}$.

P. 216, Ex. 131. — 1. $\frac{1}{3}$, 45, 15. **2.** 5 vests. **3.** $3\frac{1}{10}$ rods. **4.** $66\frac{2}{3}$¢. **5.** $4\frac{4}{5}$ A. **6.** \$$227\frac{1}{2}$. **7.** \$200. **8.** \$35.

P. 217. — **9.** 5. **10.** 40 lots. **11.** $5\frac{1}{4}$ mo. **12.** $10\frac{2}{5}$ bu. **13.** 612 lb. **14.** 48. **15.** 60. **16.** \$$14\frac{2}{5}$, \$$9\frac{3}{5}$. **17.** 16. **18.** $1\frac{1}{6}$ da., $3\frac{1}{5}$ da.

P. 218. — 19. $12\frac{1}{2}$¢. **20.** $166\frac{1}{2}$ lb. **21.** \$7, Ans. to last. **22.** $26\frac{2}{3}$ bu., Ans. to last. **23.** 12. **24.** $\frac{5}{8}$. **25.** $\frac{7}{12}$. **29.** 24 pieces.

P. 219. — **30.** 16 times. **31.** 11 strips, 8 yd. **32.** \$2.72. **33.** $6\frac{52}{70}$. **34.** $217\frac{4}{5}$ lb. **35.** \$6.96.

P. 221, Ex. 133. — **10.** $41\frac{2}{3}$. **11.** $69\frac{9}{11}$. **12.** $96\frac{2}{13}$. **13.** $21\frac{3}{17}$. **14.** $23\frac{1}{7}$. **15.** $14\frac{1}{6}$. **16.** 22. **17.** $26\frac{2}{3}$. **18.** 20. **19.** 30. **20.** 21. **21.** 18. **22.** 6. **23.** 10. **24.** 40. **25.** 32. **26.** 21. **27.** 11.

P. 222, Ex. 134. — **5.** 144 apple, 128 plum, 112 pear. **6.** \$3281.25 loss. **7.** \$760.

P. 231, Ex. 143. — **6.** .42736. **7.** 1.259153. **8.** 1.605163. **9.** 2.502444.

P. 232. — **10.** 20.182. **11.** 322.0565. **12.** 1159.3392. **13.** 31711.11676.

P. 234, Ex. 146. — 1. .97912. **2.** 1.00415. **3.** 45.3497. **4.** 1.7689. **5.** 24.065. **6.** .21479. **7.** 32.877. **8.** .0407. **9.** 2.644. **10.** .1072. **11.** .947525.

P. 235. — **12.** .024996. **13.** 24.75. **14.** .099. **15.** .107. **17.** 485.8263.

P. 236, Ex. 147. — **2.** .04125. **3.** .084. **4.** 1.082. **5.** 46.08. **6.** .015. **7.** .5. **8.** .000256. **9.** \$63.75. **10.** \$3.375. **11.** 465.375 sq. rd., 87.5 rd. **13.** 578.125 lb.

P. 240, Ex. 150. — 1. 2795. **2.** 4. **3.** 4. **4.** 8.5. **5.** 2.4. **6.** 44. **7.** 1300. **8.** 1040. **9.** .7. **10.** 5.1. **11.** 2.3.

P. 241. — **12.** 2.3. **13.** 1120. **14.** .064. **15.** 7000. **16.** 11.2. **17.** 800. **18.** 70. **19.** .079. **20.** .44.

P. 241, Ex. 151. — 1. 9 hr. **2.** 13 bu. **3.** 72 lots. **4.** 12 mo. **5.** 10 wk. **6.** \$$3.62\frac{1}{2}$. **8.** 50 yd. **9.** 34 min.

P. 244, Ex. 154. — **2.** \$3.74.

P. 245. — **3.** \$8.49. **4.** \$8.71. **5.** \$7.50. **6.** \$2.30.

P. 249, Ex. 157. — **16.** 151 yd. 2 ft., Ans. to last. **17.** 108 yd. 1 ft., Ans. to last. **18.** 32 ft. 2 in., Ans. to last. **19.** 77 ft., Ans. to last.

P. 250. — 20. 148.5 ft., Ans. to last. **21.** 44 yd., Ans. to last. **23.** $7\frac{7}{11}$ rd., Ans. to last. **25.** 12 rd., Ans. to last. **26.** 1080 rd., Ans. to last. **27.** 1720 rd., Ans. to last. **29.** 82.

P. 252, Ex. 158. — 15. 115 pt. **16.** $64\frac{1}{4}$ gal.

P. 257, Ex. 160. — 1. 5184 cu. in. **2.** 486 cu. ft. **3.** $\frac{1}{2}$. **4.** $\frac{1}{3}$. **5.** 12 cu. ft. **6.** 24 cu. ft. **7.** 112 cu. ft., Ans. to last. **8.** 1080 cu. ft. **9.** Floor, 120 sq. ft.; end, 90 sq. ft.; side, 108 sq. ft. **10.** 108 sq. ft.

P. 258, Ex. 161. — 8. 30.37\frac{1}{2}$, Ans. to last.

P. 259. — 10. 2400 rd. **11.** 14 gal., Ans. to last. **12.** 80¢, Ans. to last. **13.** $2.40. **14.** 3360 rd. **18.** $\frac{7}{18}$. **19.** $\frac{1}{2}$.

P. 260. — 20. $\frac{1}{4}$.

P. 260, Ex. 162. — 1. 296 oz., Ans. to last. **3.** 65¢. **4.** $25\frac{3}{4}$ cwt., Ans. to last.

P. 261. — 6. $\frac{1}{4}$, Ans. to last. **7.** $18. **8.** $3.75. **9.** 190 lb.

P. 262, Ex. 163 — 3. Cost, $2.40; gain, 44¢. **4.** $2.76. **5.** $83.75. **6.** 40 wk. **7.** 40 wk.

P. 265, Ex. 168. — 1. 31 pk. **2.** 44 qt. **3.** 74 ft. **4.** 739 ft. **5.** 10 lots; 4 rd.

P. 266. — 9. $\frac{5}{9}$ cut off; $\frac{4}{9}$ remaining.

P. 269, Ex. 171. — 9. 19 bu. 2 pk. **10.** 32 bu. 1 pk. **11.** 373 bu. 2 pk. **12.** 44 bu. 2 pk. **13.** 13 pk. 3 qt. **14.** 5 pk. 1 qt. **15.** 6 pk. 3 qt. **16.** 12 pk. 4 qt. **17.** 11 bu. 1 pk. **18.** 9 yd. **19.** 16 rd. 3 yd., 1 ft. 6 in. **20.** 12 lb. 1 oz. **21.** 14 lb. 3 oz. **22.** From A to C, 4 hr. 18 min; arrives at C 11 : 33 A.M. **23.** 62 ft. 6 in. **24.** 5 yd. 2 ft. 3 in.

P. 270. — 25. 19.5 cd., Ans. to last. **26.** 36 A. 20 sq. rd.

P. 270, Ex. 172. — 2. $16\frac{3}{4}$ bu. **3.** 15 qt. **4.** $10\frac{1}{4}$ gal. **5.** $14\frac{1}{6}$ ft. **6.** 13 yd. **7.** $8\frac{1}{6}$ hr. **8.** $17\frac{5}{6}$ hr. **9.** $19\frac{7}{12}$ da. **10.** $43\frac{19}{36}$ sq. ft. **11.** $21\frac{3}{8}$ A.

P. 270, Ex. 173. — 2. 3 ft. 1 in. **3.** 15 gal. 1 qt. **4.** 15 mi. 38 rd.

P. 271. — 6. 4 ft. 9 in. **7.** 4 yd. 2 ft. **8.** 16 hr. 35 min. **9.** 2 lb. 14 oz. **10.** 14 gal. 3 qt. **11.** $30\frac{1}{4}$ A. **12.** 5 A. in piece; $2\frac{3}{4}$ A. left. **13.** 120 sq. yd. in garden; 120 sq. yd. in remaining portion.

P. 272, Ex. 174. — 2. 11 pk. **3.** 40 pk. 5 qt. **4.** 80 pk. 6 qt.

P. 273. — **5.** 11 ft. **6.** 25 ft. **7.** 3208 yd. 1 ft. **8.** 10 gal. 2 qt. **9.** 27 hr. 5 min.; 2 hr. 55 min. **10.** $2.36.

P. 274, Ex. 175. — **2.** 3 bu. $2\frac{3}{4}$ pk. **3.** 2 bu. 3 pk. 7 qt. **4.** 6 yd. $1\frac{1}{4}$ ft. **5.** 3 ft. 4 in. **6.** 2 lb. 1 oz. **7.** 2 mi. 137 rd. **8.** 6 mi. 220 rd. **9.** 72 badges; 6 in. **10.** 1 lb. $5\frac{1}{3}$ oz.

P. 280, Ex. 180. — **2.** 15 plum; 60 apple.

P. 281. — **3.** 2¢, 10¢; 2¢, 6¢. **4.** $15 selling price. **5.** 70 grow; 10 fail. **6.** 36. **7.** 19; 95%. **8.** 25 bu. spoiled. **9.** $99; $11.

P. 282. — **10.** 40.

P. 282, Ex. 181. — **3.** 25%. **4.** 75%. **5.** 80%. **6.** $66\frac{2}{3}$%. **7.** $66\frac{2}{3}$%. **8.** 75%. **9.** $66\frac{2}{3}$%. **10.** 60%. **11.** 50%. **12.** 40%.

P. 283. — **13.** $33\frac{1}{3}$%. **14.** 25%. **15.** $33\frac{1}{3}$%. **16.** 40%. **17.** 40%. **18.** 25%. **19.** 50%. **20.** 80%. **21.** 75%. **22.** 75%. **23.** $33\frac{1}{3}$% sold. **24.** 25%, Ans. to last. **25.** 75%. **26.** $83\frac{1}{3}$% sold. **27.** $12; $108. **28.** 32.

P. 284. — **29.** $40. **31.** 75%; $\frac{6}{8}$ or $\frac{3}{4}$.

P. 285. — **34.** 90%. **35.** $71\frac{3}{7}$%. **36.** $88\frac{8}{9}$%. **37.** $85\frac{5}{7}$%. **38.** 20%. **39.** 50%. **40.** 75%. **41.** $16\frac{2}{3}$%. **42.** $83\frac{1}{3}$%. **43.** 40%. **44.** $31\frac{1}{4}$%. **45.** $37\frac{1}{2}$%.

P. 287, Ex. 182. — **15.** 400. **16.** 200. **17.** 400.

P. 288. — **18.** 200. **19.** 200. **20.** 700. **21.** 500. **23.** $271\frac{3}{7}$. **24.** 1600. **25.** $2083\frac{1}{3}$. **26.** 312.5. **27.** $54\frac{1}{6}$. **28.** 585.

P. 290, Ex. 183. — **3.** $36.75. **4.** $15. **5.** $25.20. **6.** $194.13. **7.** $28.60. **8.** $15.60. **9.** $20.40. **10.** $34.80.

P. 293. — **26.** Int. $47.88; Amt. $227.88. **27.** Int. $34.43; Amt. $199.43. **28.** Int. $83.03; Amt. $307.03. **29.** Int. $49.28; Amt. $217.28.

P. 294, Ex. 184. — **2.** 4 yr. 2 mo. 27 da. **3.** 3 yr. 10 mo. 29 da. **4.** 4 yr. 1 mo. 27 da. **5.** Int. $27.48; Amt. $267.48. **6.** $171.80.

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